Final-Offer Arbitration - A Look at Multi-Issue and Multi-Player Extensions

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Negotiation and Arbitration

Two parties (e.g. workers union and the company) enter into negotiations for a new contract. If the negotiations fail, the union can threaten a strike. A strike is mutually costly and motivates both parties to make concessions.

But what if a strike is not possible (Police, fire departments)? Compulsory arbitration is a typical solution.

Problem

Is bargaining compatible with compulsory arbitration?
(Stevens, 1966)

- Conventional arbitration tends to split the difference (compromise).
- This leads to a chilling effect and less desirable outcomes than could be negotiated

Solution: Final-Offer Arbitration

Final-Offer Arbitration (Stevens, 1966): The arbitrator examines the final offers of both parties and must pick one with no compromise.

Parties must find a natural balance between appealing to the judge's sense of fairness (moderate offer) and the desire for a big win (extreme offer).

An early example: The Trial of Socrates

After being found guilty of moral corruption and impiety by a jury of 500 men, Socrates and his prosecutor each proposed a punishment: a fine of 3000 drachmae or death. The jury voted.

FOA In Practice

Also known as **Pendulum** or **Baseball Arbitration**.

A variant is **MEDLOA** (Mediation with Last Offer Arbitration)

- Adopted in many states (Michigan, Wisconsin 1970s) in the public sector (e.g. police, firefighters)
- Major League Baseball after 1972 strike
- Chile's 1979 Labor Reform
- Railway shipping in Canada

The Game Model

- The judge chooses some fair value ξ and keeps it in mind.
- Player I (the minimizer, e.g. company) and Player II (the maximizer, e.g. union) present their final offers x_1, x_2 to the judge.
- Whichever final offer is closer to ξ is the settlement.

Further assumptions

- As far as players are concerned, the fair settlement is chosen randomly from a distribution F with density function f.
- F is common knowledge.
- WLOG, the median of the distribution is 0.
- The game is zero-sum.

Game Rewards

The payment made by Player I to Player II is

$$K(x_1, x_2|\xi) = \begin{cases} x_1 & \text{if } |x_1 - \xi| < |x_2 - \xi| \\ x_2 & \text{if } |x_1 - \xi| > |x_2 - \xi| \end{cases}$$

If $|x_1 - \xi| = |x_2 - \xi|$, the payment is x_1 or x_2 with equal probability.

Assuming $x_1 < x_2$, the expected payoff may be written

$$K = x_1 P\left(\xi < \frac{x_1 + x_2}{2}\right) + x_2 P\left(\xi > \frac{x_1 + x_2}{2}\right)$$

or equivalently

$$K = x_2 + (x_1 - x_2) F\left(\frac{x_1 + x_2}{2}\right)$$

Minimax Theorem

A pair of strategies x_1^* and x_2^* are said to be optimal if

$$K(x_1, x_2^*) \geq K(x_1^*, x_2^*) \geq K(x_1^*, x_2)$$

for all x_1, x_2.

The **minimax theorem** (von Neumann, 1928) states that all zero-sum games have optimal strategies (though they may be pure or mixed).

Brams-Merrill Theorem (1983)

Theorem

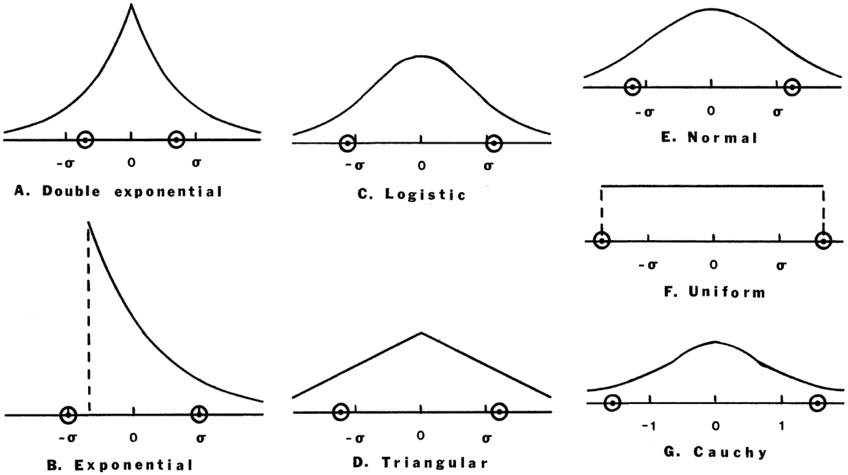
(1) If $f'(0)$ exists and $f(0) > 0$, then locally optimal strategies are

$$x_1^* = -\frac{1}{2f(0)} \quad \textit{and} \quad x_2^* = \frac{1}{2f(0)}.$$

(2) If f is "sufficiently concentrated at the median", then these represent the unique globally optimal strategy pair.

Brams and Merrill also provide a weaker condition for global optimality.
Both Normal and Uniform distributions satisfy the second condition.

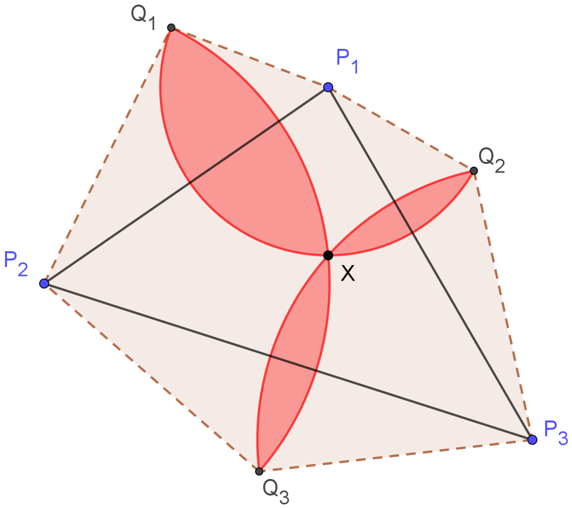
Divergence of Global Optimal Pure Strategies



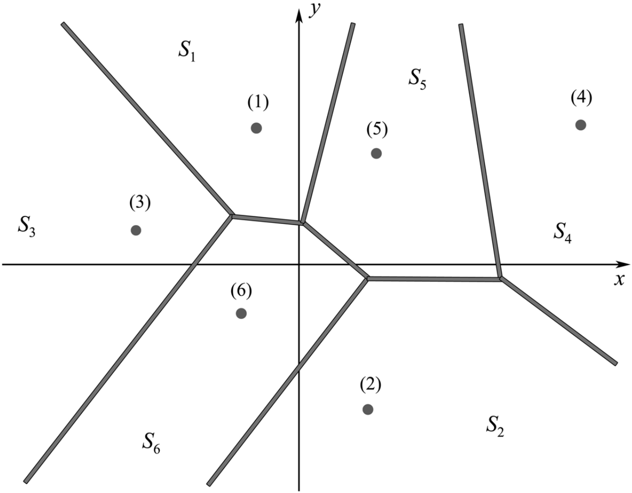
(Brams-Merrill, 1983)

Related Problems

Optimal Location of Candidates in Ideological Space (Owens, Shapley 1989)



Game-Theoretic Models of Tender Design (Mazalov, Tokareva, 2014)



Multiple-Issue FOA

When more than one issue is being arbitrated, two major variants of FOA have been used (Farber, 1980):

- **Issue by Issue:** Each party submits a vector of final offers and the arbitrator is free to compose a compromise by selecting some offers from each party
- **Whole Package:** Both parties submit a vector of final offers and the arbitrator must choose one or the other in its entirety

Challenges in Extending the Model

- How do we compare players' valuation of settlement bundles?
- How do we model the uncertainty of the arbitrator's opinion?
- How does the arbitrator measure "closeness"?
- How do we handle qualitative issues in dispute?
- What if separate quantitative issues are not fungible?
- Modeling risk aversion?
- Extending to multiple players?

The Multi-Issue Game Setting

- Players I and II present final-offers $\mathbf{a}, \mathbf{b} \in \mathbb{R}^d$
- Judge selects $\boldsymbol{\xi} \sim F$ as an ideal fair settlement.
- F is common knowledge.
- Judge uses reasonableness function $R(\mathbf{x}, \boldsymbol{\xi}) : \mathbb{R}^d \times \mathbb{R}^d \to \mathbb{R}$ to compare final-offers.
- Game is zero-sum.
- Payoff is

$$K(\mathbf{a}, \mathbf{b} | \boldsymbol{\xi}) = \begin{cases} \sum_i a_i & R(\mathbf{a}, \boldsymbol{\xi}) > R(\mathbf{b}, \boldsymbol{\xi}) \\ \sum_i b_i & R(\mathbf{a}, \boldsymbol{\xi}) < R(\mathbf{b}, \boldsymbol{\xi}) \end{cases}$$

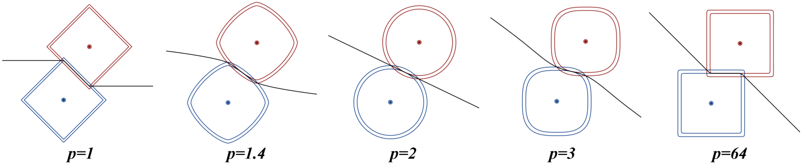
Choice of F and R

Distribution	
Normal	$\boldsymbol{\xi} \sim \mathcal{N}(\mu, \Sigma)$ (WLOG, assume $\mu = \mathbf{0}$)
Uniform	$\boldsymbol{\xi} \sim \mathit{Unif}\left(\times_{j=1}^{d}[-\alpha_j, \alpha_j]\right)$, where $\alpha_j > 0$

Criterion	Reasonableness Function
Net Offer	$R_{NO}(\mathbf{x}, \boldsymbol{\xi}) = -\left\lvert \sum_{j=1}^{d} x_j - \xi_j \right\rvert$
$\mathbf{L}_1$	$R_{L_1}(\mathbf{x}, \boldsymbol{\xi}) = -\sum_{j=1}^{d} \lvert x_j - \xi_j \rvert$
$\mathbf{L}_\infty$	$R_{L_\infty}(\mathbf{x}, \boldsymbol{\xi}) = -\max_j \{\lvert x_j - \xi_j \rvert\}$
$\mathbf{L}_p$	$R_{L_p}(\mathbf{x}, \boldsymbol{\xi}) = -\sum_{j=1}^{d} \lvert x_j - \xi_j \rvert^p$
$\mathbf{L}_2$	$R_{L_2}(\mathbf{x}, \boldsymbol{\xi}) = -\sum_{j=1}^{d} (x_j - \xi_j)^2$
Mahalanobis	$R_M(\mathbf{x}, \boldsymbol{\xi}) = -(\mathbf{x} - \boldsymbol{\xi})' \Sigma^{-1} (\mathbf{x} - \boldsymbol{\xi})$

$2NL_p$ Circles

$$D_{L_p}(\mathbf{x}, \mathbf{y}) = \left(\sum_{i=1}^{d} |x_i - y_i|^p \right)^{1/p}$$



"Circles" and Midset curves under Minkowski metrics (L_p)

$2NL_1$, $2NL_\infty$, $2NL_p$

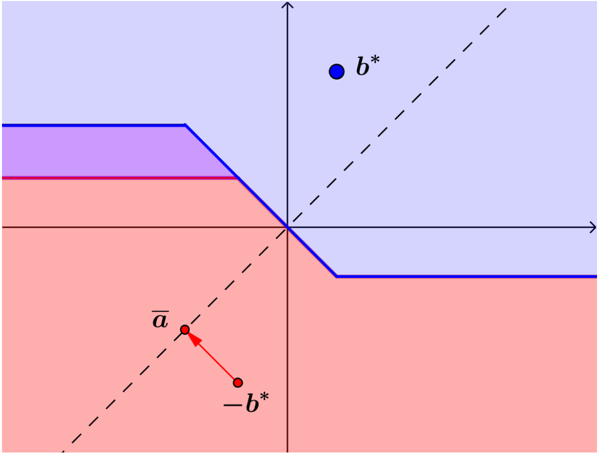
Theorem

In $2NL_1$, $2NL_\infty$ or $2NL_p$ if pure optimal strategies exist for Players $i = 1, 2$ then they are given by

$$(x_i^*, y_i^*) = \left((-1)^i x^*, (-1)^i x^*\right),$$

where $x^* = \dfrac{\sqrt{2\pi(\sigma_x^2 + 2\rho\sigma_x\sigma_y + \sigma_y^2)}}{4}$.

$2NL_1$ Proof Sketch



As it is sub-optimal for either player to choose a pure strategy off the line $y = x$, the game reduces to the one-dimensional case.

$2NL_2$ Local Optimality

Theorem

In $2NL_2$, suppose $\rho > \max\left\{-\frac{\sigma_x^2+3\sigma_y^2}{4\sigma_x\sigma_y}, -\frac{3\sigma_x^2+\sigma_y^2}{4\sigma_x\sigma_y}\right\}$. The pure strategy pair in the previous theorem is locally optimal.

Because $Mid_{L_2}[\mathbf{a}, \mathbf{b}]$ is a straight line, we can essentially reduce the dimension of F.

$2NL_2$ Payoff Function

Let $(\xi, \eta) \sim \mathcal{N}(\mathbf{0}, \Sigma)$ be the opinion of the arbitrator, where

$$\Sigma = \begin{bmatrix} \sigma_x^2 & \rho\sigma_x\sigma_y \\ \rho\sigma_x\sigma_y & \sigma_y^2 \end{bmatrix}$$

$C_1(\mathbf{a}, \mathbf{b}) = \{(x, y) | (x_1 - x)^2 + (y_1 - y)^2 < (x_2 - x)^2 + (y_2 - y)^2\}$
and $C_2(\mathbf{a}, \mathbf{b})$ defined similarly.
$P_i = P\big((\xi, \eta) \in C_i(\mathbf{a}, \mathbf{b})\big)$
Assuming $\mathbf{a} \neq \mathbf{b}$, the expected payoff

$$K(\mathbf{a}, \mathbf{b}) = (x_1 + y_1)P_1 + (x_2 + y_2)P_2$$

may be written

$$K(\mathbf{a}, \mathbf{b}) = (x_2 + y_2) + (x_1 + y_1 - x_2 - y_2)P_1.$$

$2NL_2$ Local Optimality Proof Overview cont.

Express P_1 one-dimensionally:

$$\begin{aligned} P_1 &= P\big((x_1-\xi)^2+(y_1-\eta)^2<(x_2-\xi)^2+(y_2-\eta)^2\big) && (1)\\ &= P\left((x_2-x_1)\xi+(y_2-y_1)\eta<\frac{x_2^2+y_2^2-x_1^2-y_1^2}{2}\right) && (2)\\ &= P(Z<z) && (3)\end{aligned}$$

where

$$z=\frac{x_2^2+y_2^2-x_1^2-y_1^2}{2\sqrt{(\mathbf{b}-\mathbf{a})^T\Sigma(\mathbf{b}-\mathbf{a})}} \tag{4}$$

we may write

$$K(\mathbf{a},\mathbf{b})=(x_2+y_2)+(x_1+y_1-x_2-y_2)\Phi(z) \tag{5}$$

where $\Phi(z)$ is the standard Gaussian cdf.

$2NL_2$ Local Optimality Proof Overview cont.

By solving the system of first-order equations

$$\frac{d}{dx_1}K|_{\mathbf{a}^*,\mathbf{b}^*} = \frac{d}{dy_1}K|_{\mathbf{a}^*,\mathbf{b}^*} = \frac{d}{dx_2}K|_{\mathbf{a}^*,\mathbf{b}^*} = \frac{d}{dy_2}K|_{\mathbf{a}^*,\mathbf{b}^*} = 0$$

we arrive at the unique $\mathbf{a}^*, \mathbf{b}^*$ given in the theorem. It is *straightforward* to verify that the second order condition holds provided

$$\rho > \max\left\{-\frac{\sigma_x^2 + 3\sigma_y^2}{4\sigma_x\sigma_y}, -\frac{3\sigma_x^2 + \sigma_y^2}{4\sigma_x\sigma_y}\right\}.$$

$2NL_2$ Global Optimality

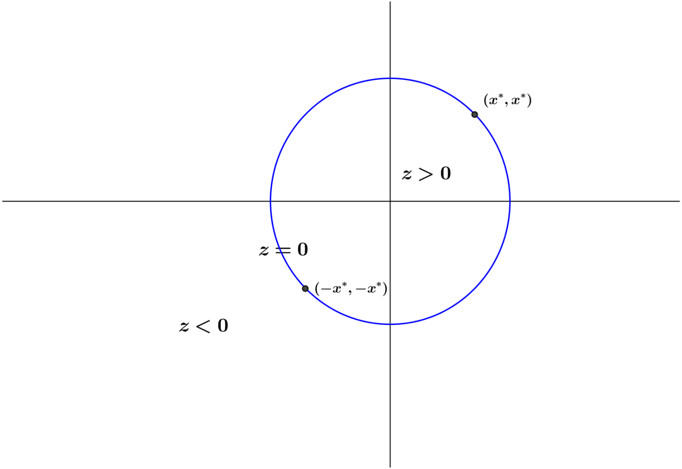
Theorem

If $\rho > 0$, the solution points $\mathbf{a}^*, \mathbf{b}^*$ given in the previous theorem are globally optimal.

In other words, $K(\mathbf{a}, \mathbf{b}^*) \geq 0 \, \forall \mathbf{a} \in \mathbb{R}^2$, with equality only when $\mathbf{a} = \mathbf{a}^*$.

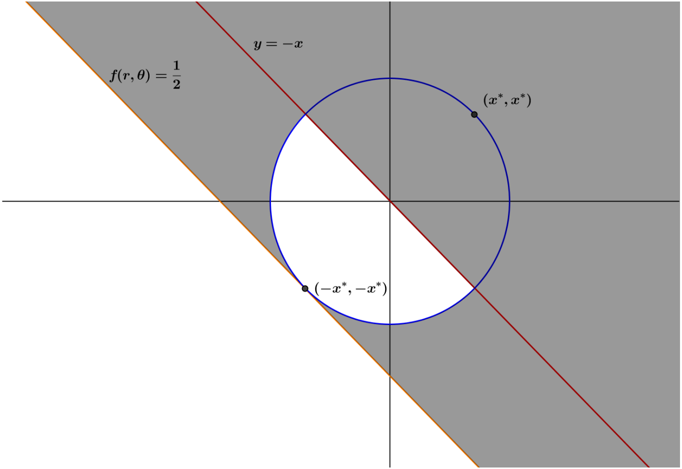
Thus players need not consider mixed strategies. The proof relies on a geometric interpretation of the players' strategies.

$2NL_2$ Global Optimality Proof Overview



$$z = \frac{2x^{*2} - x_1^2 - y_1^2}{2\sqrt{(\mathbf{b}^* - \mathbf{a})^T \Sigma (\mathbf{b}^* - \mathbf{a})}}$$

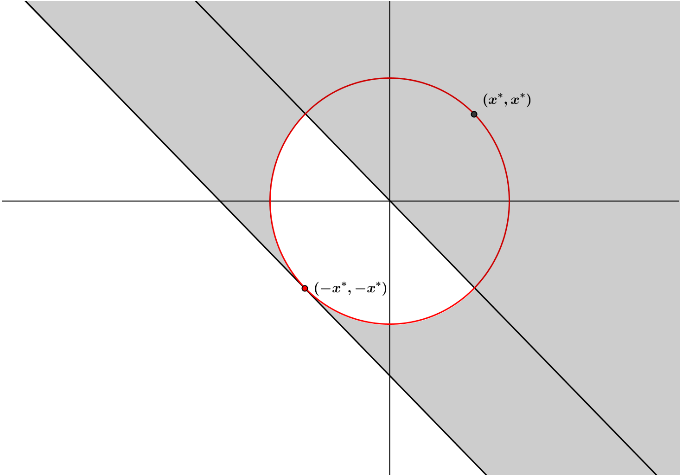
$2NL_2$ Global Optimality: Proof Overview cont.



If $K(\mathbf{a}, \mathbf{b}^*) \leq 0$ then $x_1 + y_1 < 0$ and either

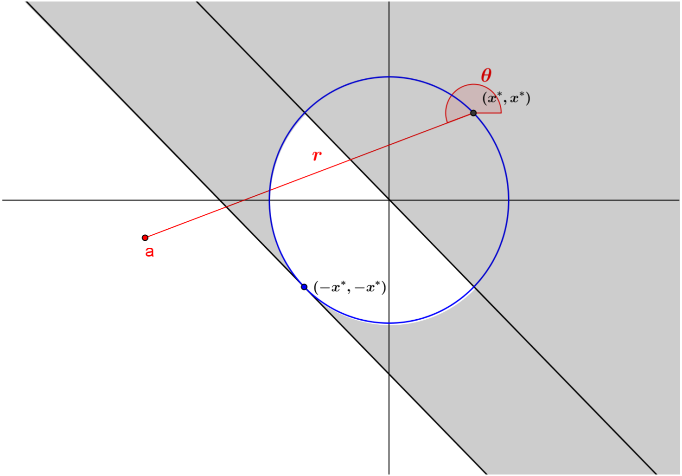
$$x_1^2 + y_1^2 < 2x^{*2} \quad \text{or} \quad x_1 + y_1 \leq -2x^*.$$

$2NL_2$ Global Optimality: Proof Overview cont.



$$K(\mathbf{a}, \mathbf{b}^*) = 2x^* + \frac{1}{2}(x_1 + y_1 - 2x^*) = 2x^* + x_1 + y_1 \geq 0$$

when $z = 0$, with equality only when $\mathbf{a} = (-x^*, -x^*)$.



Against Player II's strategy $\mathbf{b}^* = (x^*, x^*)$, any pure strategy $\mathbf{a} = (x_1, y_1)$ may be represented as

$$\mathbf{a}(r, \theta) = (x^* + r\cos\theta, x^* + r\sin\theta).$$

$2NL_2$ Global Optimality: Polar Representation

Letting $t(\theta) = -\cos\theta - \sin\theta$,

$$K(\mathbf{a}, \mathbf{b}^*) = 2x^* - rt(\theta)\Phi(z)$$

So $K < 0$ is equivalent to

$$\Phi(z) > \frac{2x^*}{rt(\theta)} = f(r, \theta)$$

$2NL_2$ Global Optimality: Two Tricks

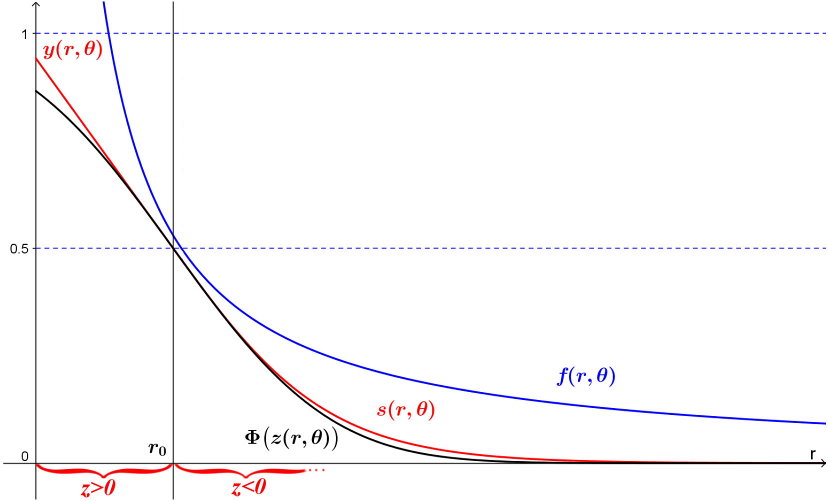
To avoid the difficulties inherent in $\Phi(z)$, we use two tricks:
(1) For $z < 0$, the normal cdf is bounded by the sigmoidal

$$\Phi(z) < \frac{1}{1 + \exp\left(-\sqrt{\frac{8}{\pi}} z\right)}$$

(2) For $z > 0$, by its concavity, $\Phi(z) < y(z)$, the line tangent at $z = 0$

$2NL_2$ Global Optimality: Proof Overview cont.

For fixed $\theta \in \left[\frac{3\pi}{4}, \frac{7\pi}{4}\right]$:



It may come as no surprise that the arbitrated outcome using whole Package has a higher variance than Issue-By-Issue.

Theorem

The expected payoff is zero under both Issue-by-Issue and Whole-Package variants. If both player play optimally then the variances of the awards are $\frac{\pi}{2}(\sigma_x^2 + \sigma_y^2)$ *and* $\frac{\pi}{2}(\sigma_x^2 + 2\rho\sigma_x\sigma_y + \sigma_y^2)$ *respectively.*

$2UL_2$ - Globally Optimal Pure Strategies

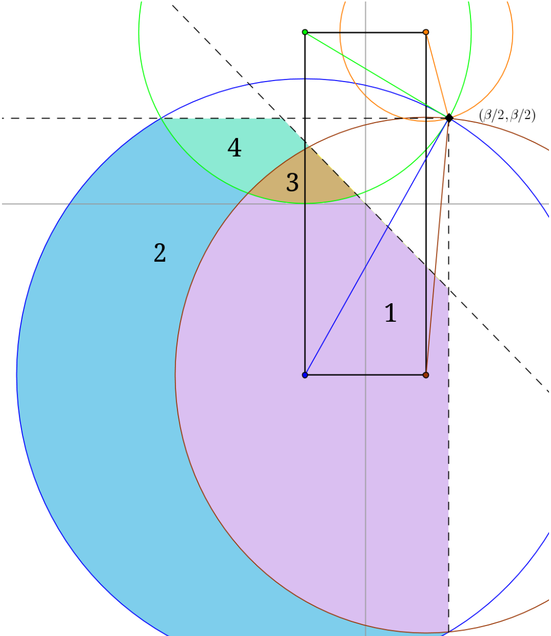
Suppose $\boldsymbol{\xi}$ is drawn uniformly at random from $\Xi := [-\alpha, \alpha] \times [-\beta, \beta]$, where WLOG $0 < \alpha \leq \beta$, and the judge uses the L_2 metric.

Theorem

In $2UL_2$, the strategy pair $\mathbf{a}^* = \left(-\frac{\beta}{2}, -\frac{\beta}{2}\right), \mathbf{b}^* = \left(\frac{\beta}{2}, \frac{\beta}{2}\right)$ is the unique globally optimal strategy pair.

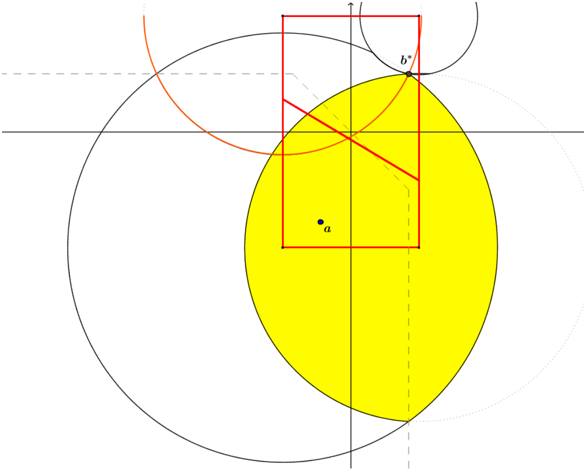
To prove this, we let Player II play $\mathbf{b}^*$ and show that the expected payoff is minimized only when Player I plays $\mathbf{a}^*$.

$2UL_2$ Proof - Cases



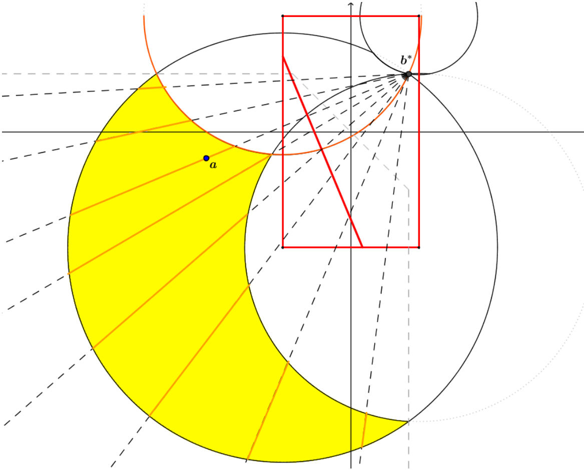
$2UL_2$ Proof Case 1

In the first case, we can show directly that the payoff function is minimized only at $\mathbf{a} = \mathbf{a}^*$, which lies in this region.



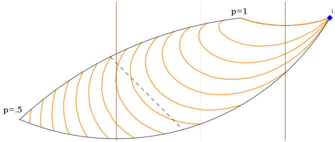
$2UL_2$ Proof Case 2

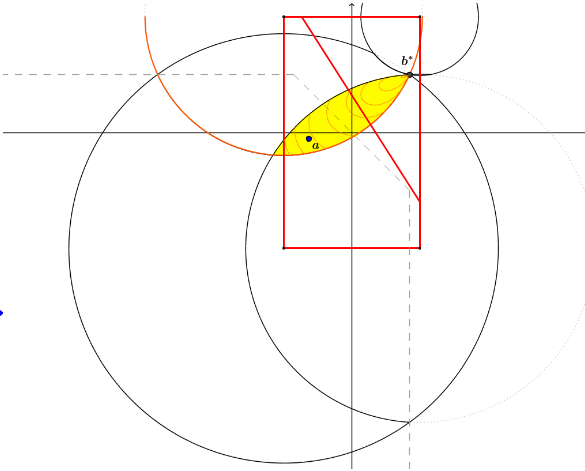
Here we parameterize the strategy of Player I along line segments by slope m and $\lambda \in [0, 1]$, and show that the payoff function is a decreasing function of λ, and on the boundary the payoff is positive.



$2UL_2$ Proof Case 3

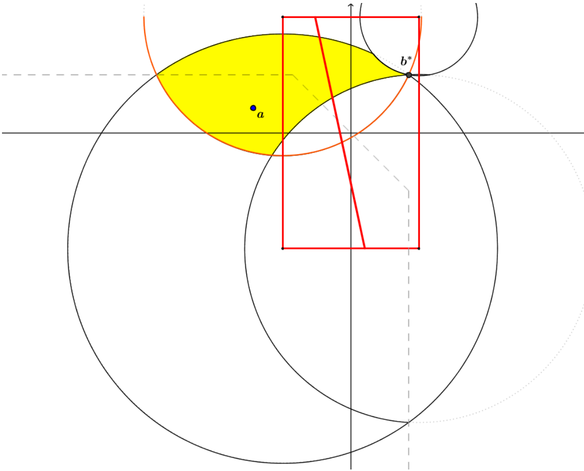
We parameterize the strategy by $p \in [.5, 1]$ (i.e. P_1) and $\bar{x} \in [0, 2\alpha]$ (the length of the upper boundary of C_1) to show there is no local minima.





$2UL_2$ Proof Case 4

The final case is handled directly; it is shown by the first order condition that no minimum to the payoff function exists in this region.



Nash Equilibrium

In non-zero sum games, the most popular solution concept is the Nash equilibrium.

Let $x_i \in S_i$ be the strategy of Player i from strategy space i. The reward functions are $K_i(x_1, \ldots, x_n)$. A strategy profile $x_1^*, \ldots, x_n^*$ is a Nash equilibrium if for each player i,

$$K_i(x_i, \mathbf{x}_{-i}^*) \leq K_i(\mathbf{x}^*)$$

In other words, knowing the strategy $x_1^*, x_2^*, \ldots, x_n^*$ of the other players, player i has no incentive to deviate from x_i^*.

Every continuous game with compact strategy spaces and continuous utility functions are guaranteed a Nash equilibrium (pure or mixed).

Interpleader

In 1972, Ferdinand Marcos, then the President of the Republic of the Philippines, deposited approximately $2 million with Merrill Lynch in New York City. That money sat in a Merrill Lynch account for the next thirty-odd years, growing to approximately $33.8 million worth of cash and securities. By 2000, a number of claimants to Marcos's estate had come knocking, so Merrill Lynch filed an interpleader to determine who should get the money.

https://www.casemine.com/judgement/us/591465abadd7b049342906e8

A dispute such as this between 3 or more parties may be modeled by an N-player arbitration game.

Generalize to N players

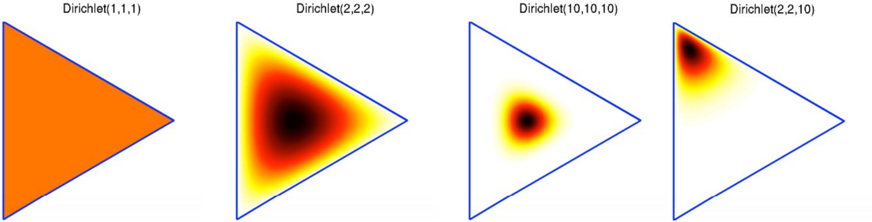
A unit must be split between the players. Each player i chooses a vector $\mathbf{x}^i \in \Delta^N$, where $\Delta^N = \{\mathbf{x} | \mathbf{x} \in \mathbb{R}^N, \sum x_i = 1, \mathbf{x} \geq 0\}$. Suppose each player i supplies evidence of strength $\lambda_i \geq 0$ in her favor to the judge. If $\lambda_i = 0$ then the player has supplied no evidence in her favor. Suppose that based on this evidence the judge decides on a fair split of the unit award. Let $\boldsymbol{\xi} = (\xi_1, \xi_2, \ldots, \xi_N)$ be the fair split, where $\boldsymbol{\xi} \in \Delta^N$.

Dirichlet Distribution

Assume that it is common knowledge among the players that $\boldsymbol{\xi}$ will be drawn from a Dirichlet distribution with parameter $\boldsymbol{\alpha} = (\alpha_1, \ldots, \alpha_N)$, where $\alpha_i = \lambda_i + 1$.. The density function is

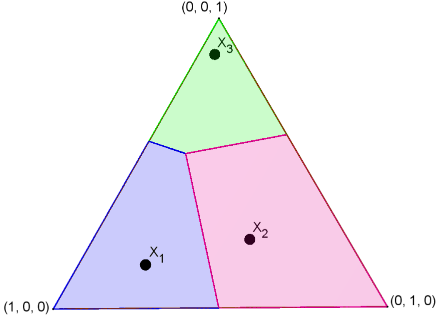
$$f(\mathbf{x}) = \frac{\prod_{i=1}^{N} x_i^{\lambda_i}}{B(\boldsymbol{\alpha})}$$

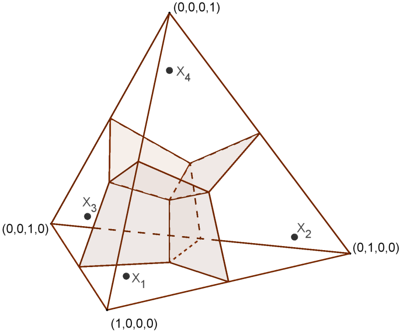
Where $B(\boldsymbol{\alpha})$ is a normalizing constant.



Voronoi Cells in the Simplex

Given the N final-offers, we may partition Δ^N into N convex Voronoi cells. Call these V_i for $i = 1, \ldots, N$.





If $\lambda_i = 0$ for all players, the probability distribution is uniform over the simplex. In this case, the payoff function is

$$K_i(\mathbf{x}^1, \dots, \mathbf{x}^N) = (N-1)! \sum_{j=1}^{N} \mathbf{x}_i^j \iint_{V_j} \frac{N! \sqrt{2^N}}{\sqrt{N+1}} dV_j$$

Theorem

Let $N \geq 3$. Players $2, \ldots, N$ demand β and offer $\frac{1-\beta}{N-1}$ to the opponents. Player 1 determines to demand α. Then P_1 is maximized when Player 1 offers $\frac{1-\alpha}{N-1}$ to each other player.

This can be proven inductively, for in the $(N-1)$-simplex an equal split maximizes the volume of the $N-1$ faces adjacent to $(1, 0, \ldots, 0)$ which are $(N-2)$-simplices, and maximizes the distance from $(1, 0, \ldots, 0)$ to the opposite vertex.

The harmonic numbers H_N are defined as the sum of the inverses of the first n integers:

$$H_n = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}$$

The harmonic numbers roughly approximate the natural logarithm.

Theorem

For an N player FOA game where ξ is chosen uniformly at random, assuming the conjecture, a pure equilibrium strategy is for each player to demand $\frac{H_{N-1}}{N-1}$ for himself and offer the remaining portion equally to the other players.[1]

Relative Greed Grows Logarithmically

The relative greed of a player i demanding x_i would be

$$g_i = \frac{x_i}{\frac{1}{N}} = Nx_i$$

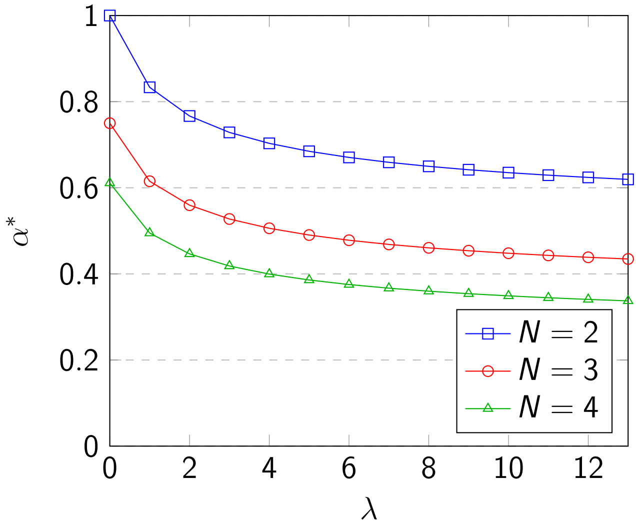
When playing the pure equilibrium strategy, the relative greed $g_i = \frac{N}{N-1} H_{N-1} \approx \ln(N-1)$. This is to say, although in equilibrium players demand less as the number of players increases, their relative greed actually increases logarithmically with the number of players.

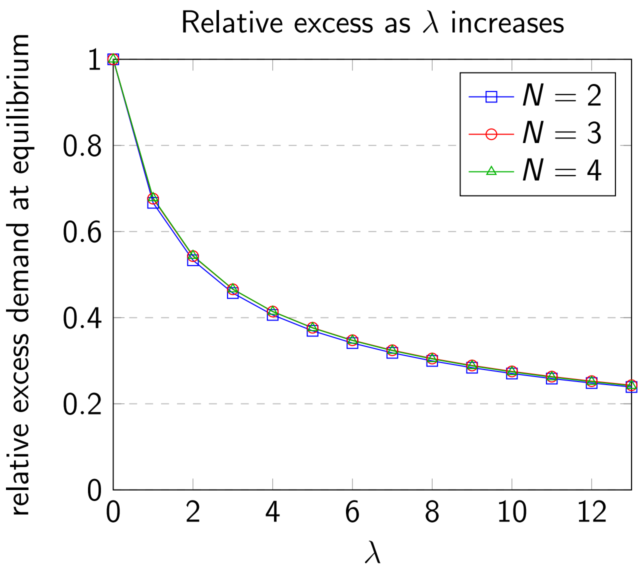
Symmetric Dirichlet Distributions

Suppose each player gives the same level of evidence in his favor. In other words, $\lambda_1 = \cdots = \lambda_N = \lambda$. It would make sense that the increasing concentration of probability at the mode, $(\frac{1}{N}, \ldots, \frac{1}{N})$ would cause the Player's equilibrium offers to converge, but is this the case? And at what rate?
In the 2 player case, the game becomes zero-sum with $\xi \sim Beta(\lambda + 1, \lambda + 1)$, we know that the optimal pure strategy for each player is to demand

$$x^* = \frac{1}{2} + \frac{\Gamma(\lambda + 1)^2 4^\lambda}{2\Gamma(2\lambda + 2)}.$$

Convergence to the mean as λ increases




Relative excess as λ increases
relative excess demand at equilibrium
λ
$N = 2$
$N = 3$
$N = 4$
0
0.2
0.4
0.6
0.8
1
0
2
4
6
8
10
12

This suggests that we may be able to approximate the Nash equilibrium for any N and λ

$$\alpha^*(N, \lambda) \approx \frac{1}{N} + \frac{\sqrt{\pi}\Gamma(\lambda+1)H_{N-1}}{2\Gamma(\lambda+\frac{3}{2})(N-1)}$$

Non-Zero Sum 2-Issue 2-Player FOA

We will not assume the issues are fungible or even in the same units. Because both players know the judge chooses a fair settlement from F, they may standardize their offers;

$$(x_i, y_i) \to \left(\frac{x_i}{\sigma_x}, \frac{y_i}{\sigma_y} \right)$$

So effectively we may assume that the judge chooses (ξ, η) from $N(\mathbf{0}, \Sigma)$ with

$$\Sigma = \left[\begin{array}{cc} 1 & \rho \\ \rho & 1 \end{array} \right]$$

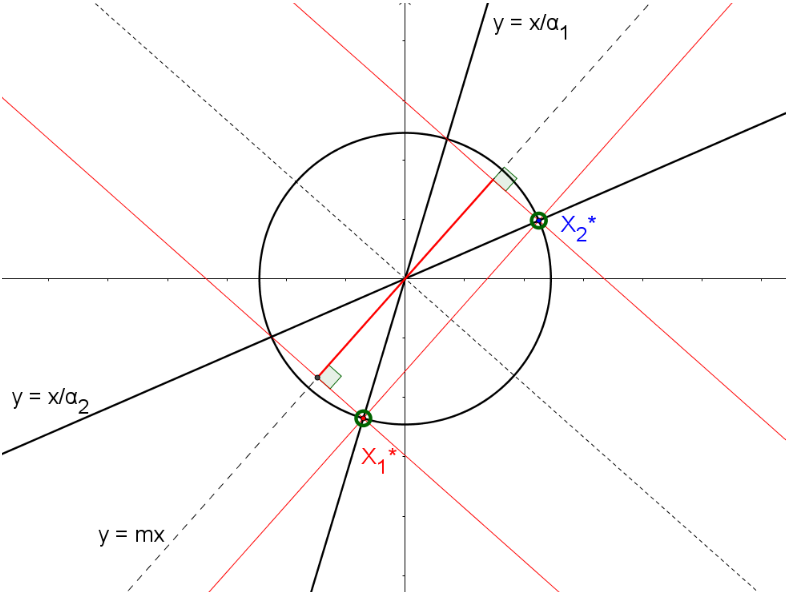
The players value a settlement (x, y) as $v_i(x, y) = \alpha_i x + \beta_i y$, where $\alpha_1, \beta_1 < 0$, $\alpha_2, \beta_2 > 0$. However, as we may scale the payoff functions without affecting the game, we may as well assume that $\beta_1 = -1$ and $\beta_2 = 1$.

Thus

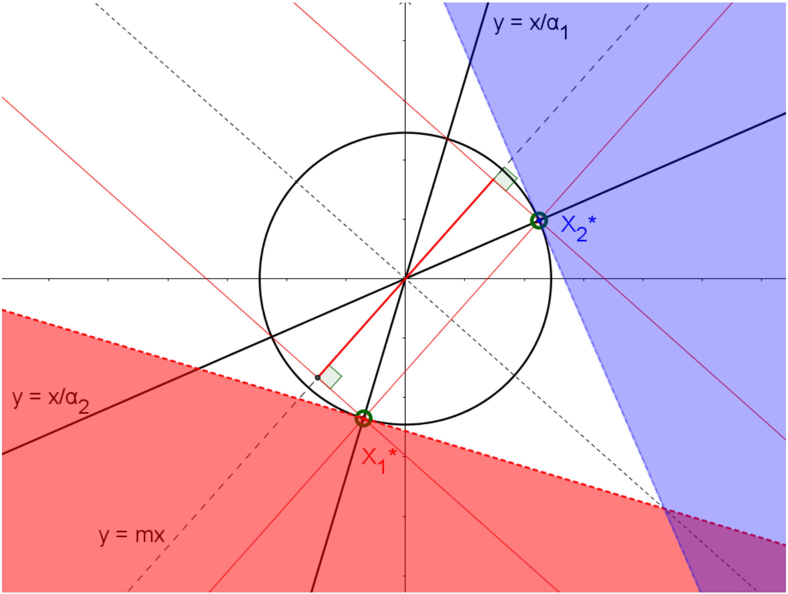
$$K_1(\mathbf{a}, \mathbf{b}) = \alpha_1 x_2 - y_2 + \big(\alpha_1(x_1 - x_2) - (y_1 - y_2)\big)\Phi(z) \quad (6)$$

$$K_2(\mathbf{a}, \mathbf{b}) = \alpha_2 x_1 + y_1 + \big(\alpha_2(x_2 - x_1) + (y_2 - y_1)\big)\Phi(-z) \quad (7)$$

Possible Pure Equilibria



Improvement through negotiation



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Thank You