Analysis of Multi-Issue Final-Offer Arbitration

Brian Powers
bpower6@math.uic.edu

UIC
UNIVERSITY OF ILLINOIS AT CHICAGO

Arbitration Methods

Should negotiations break down, arbitration by a neutral third party may resolve a dispute.

Conventional Arbitration: The arbitrator may construct a compromise between the two parties' demands

Final-Offer Arbitration: [Stevens, 1966] The arbitrator must choose between the final offers of each of the parties.

Pure Equilibria for Single-Issue Setting

Arbitrator chooses a *fair* settlement ξ from continuous, differentiable F with density f and median 0. Player I (minimizer) and II (maximizer) respectively choose final offers x_1 and x_2. Assume players are risk neutral. The payoff (from I to II) is

$$K(x_1, x_2) = x_2 + (x_1 - x_2)F\left(\frac{x_1 + x_2}{2}\right)$$

Theorem 0.1. *[Brams, Merrill, 1983]* $\left(\frac{-1}{2f(0)}, \frac{1}{2f(0)}\right)$ *is a pure equilibrium, provided $f(0) > 0$ and F has certain other properties.*

Proof. The solution points are found by taking the first derivative of K, setting equal to zero and solving the system of equations. □

Multi-Issue Final-Offer Arbitration

Two variants of FOA have been used primarily [Farber, 1980]:

- **Issue by Issue:** Each party submits a vector of final-offers and the arbitrator crafts a compromise by selecting some offers from each vector
- **Whole Package:** Both parties submit a vector of final-offers and the arbitrator must choose one or the other in its entirety

We will consider the Whole Package case.

Our Problem Setting

The arbitrator chooses a fair settlement vector ξ drawn from an $n-$dimensional Normal distribution known to both players.

$$\xi \sim N_n(0, \Sigma)$$

Players select $\mathbf{x_1}, \mathbf{x_2}$. The arbitrator selects a criterion for deciding which offer is "more reasonable", i.e. "closer" to ξ: **Nearest Net Offer**, **L_p distance ($p \geq 1$)**, **Mahalanobis distance**, **Majority of closer components (odd n)**. Furthermore, the vectors may need to be standardized.

Case 1: $n = 2$

1 Pure Strategy Equilibria

Suppose players are restricted to using pure strategies $(x_1, y_1), (x_2, y_2)$ and

$$\mathbf{\Sigma} = \begin{bmatrix} \sigma_x^2 & \rho\sigma_x\sigma_y \\ \rho\sigma_x\sigma_y & \sigma_y^2 \end{bmatrix}.$$

Let $\alpha = \sigma_x^2 + \rho\sigma_x\sigma_y$, $\beta = \sigma_y^2 + \rho\sigma_x\sigma_y$.

Theorem 1.1. *Under "Nearest Net Offer" criterion, define S_1^*, S_2^* by*

$$S_i^* = \left\{(-1)^i \frac{\sqrt{2\pi(\alpha+\beta)}}{2}(\gamma, 1-\gamma) : \gamma \in \mathrm{R}\right\}$$

Any pair (s_1^*, s_2^*) with $s_i^* \in S_i^*$ is a pure strategy equilibrium.

Theorem 1.2. *Under the following decision criteria, pure strategy equilibria are $\mathbf{x}_1^* = -\mathbf{x}_2^*$, and $\mathbf{x}_2^*$ is given by*

Decision Criterion	$\mathbf{x}_2^*$
L_p distance, $p \geq 1$	$\frac{\sqrt{2\pi(\alpha+\beta)}}{4}(1,1)$
Mahalanobis distance	$\frac{\sqrt{2\pi(\sigma_x^2\alpha^2+\rho\sigma_x\sigma_y\alpha\beta+\sigma_x^2\beta^2)}}{2(\alpha^2+\beta^2)}(\alpha,\beta)$
Standardized L_p distance	$\frac{\sqrt{\pi(1+\rho)}}{2}(\sigma_x, \sigma_y)$

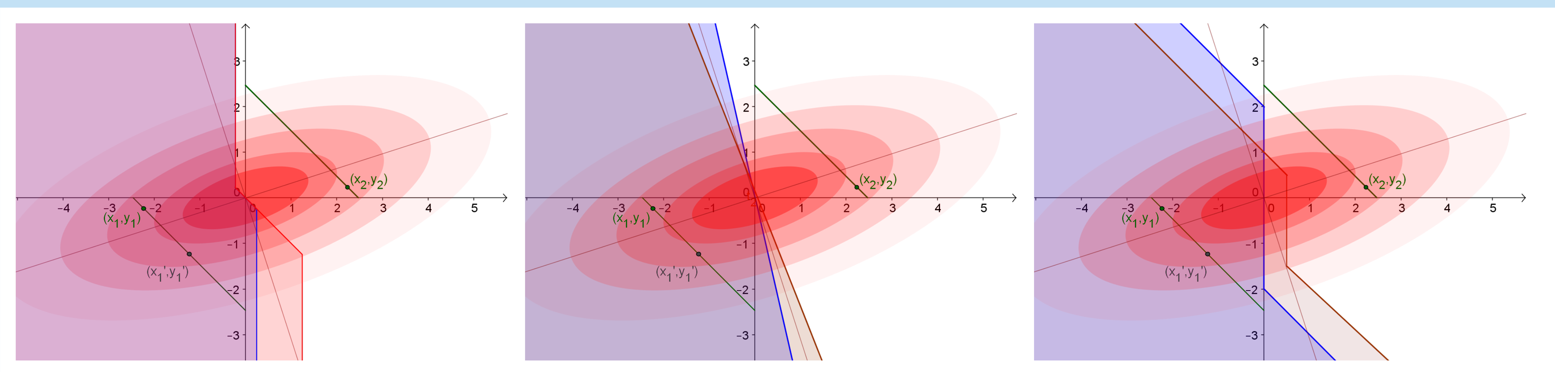

Figure 1: Graphical sketch of the proof for L_p, $p = 1, 2$ and ∞ respectively.

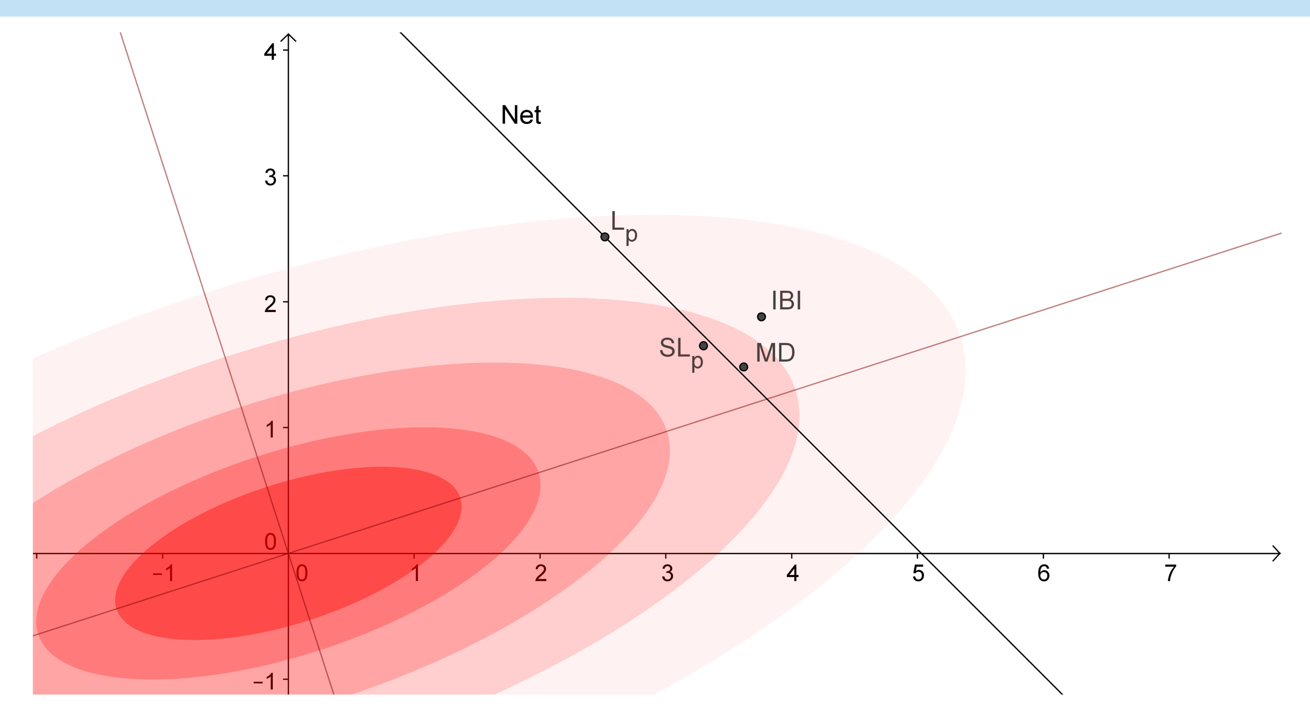

Figure 2: Solution points for Player II under: Issue by Issue (IBI), Nearest Net Offer (Net), L_p, Mahalanobis Distance (MD) and Standardized L_p (SL$_p$) with $\sigma_x = 3, \sigma_y = 1.5, \rho = .54$

Case 1: $n = 2, L_2$ Global Equilibria

Theorem 1.3. *If the judge uses L_2 criterion, the solution points for the two players given in Theorem 1.2 are locally optimal provided*

$$\rho > -\frac{\sigma_x^2 + \sigma_y^2}{2\sigma_x\sigma_y} + \frac{|\sigma_x^2 - \sigma_y^2|}{\sqrt{12}\sigma_x\sigma_y}.$$

Theorem 1.4. *If the judge uses L_2 criterion and $\sigma_1 = \sigma_2 = 1$, then the solution points for the two players given in Theorem 1.2 a global equilibrium.*

Case 2: $n \geq 2$

2 Distance Metrics as Criteria

Suppose players are restricted to using pure strategies in the n-dimensional case. Let $\sigma_\zeta^2 = \sum_i \sum_j \sigma_{ij}$

Theorem 2.1. *Under "Nearest Net Offer" criterion, define S_1^*, S_2^* as*

$$S_i^* = \left\{(-1)^i \sigma_\zeta \sqrt{\pi/2}\gamma : \gamma \in \mathbb{R}_n, \sum_{j=1}^n \gamma_j = 1\right\}$$

Any pair (s_1^*, s_2^*) with $s_i^* \in S_i^*$ is a pure strategy equilibrium.

Theorem 2.2. *Under L_1 or L_2 Distance criterion,*

$$\mathbf{x}_1^* = -\frac{\sigma_\zeta\sqrt{2\pi}}{2n}\mathbf{1}_n \quad \mathbf{x}_2^* = \frac{\sigma_\zeta\sqrt{2\pi}}{2n}\mathbf{1}_n$$

are optimal solutions for the two players.

3 Majority of Closer Components

If n is odd, the arbitrator may side with whichever player has the majority of offer components closest to ξ. If every component of a player's offer vector is finite, we say that the offer is **reasonable**.

Theorem 3.1. *Under the "Majority of Closer Components" criterion, if players strategy space is unbounded then the players are driven to unreasonable offers.*

References

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[3] Steven J Brams and Samuel Merrill III. Equilibrium strategies for final-offer arbitration: There is no median convergence. *Management Science*, 29(8):927–941, 1983.