A Few Games and Geometric Insights

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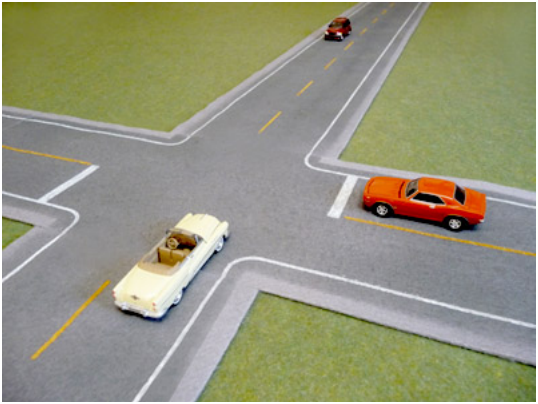
Outline

For Example

Two cars approach an intersection...



Each driver can choose to drive ("Dare") or stop ("Chicken").

The Game of "Chicken"

We have four possible outcomes of our game

$$\begin{array}{c c} & \begin{array}{cc} 2 : \textit{Dare} & 2 : \textit{Chicken} \end{array} \\ \begin{array}{c} 1 : \textit{Dare} \\ 1 : \textit{Chicken} \end{array} & \begin{pmatrix} \text{Collision} & \text{2 Passes} \\ \text{1 Passes} & \text{Both Stop} \end{pmatrix} \end{array}$$

The Game of "Chicken"

The game game with ordered pairs assigning rewards to players:

$$\begin{array}{cc} & \begin{array}{cc} D & C \end{array} \\ \begin{array}{c} D \\ C \end{array} & \begin{pmatrix} (0,0) & (7,2) \\ (2,7) & (6,6) \end{pmatrix} \end{array}$$

Each player secretly commits to a strategy, then both act simultaneously.
Finally each receives his component reward.

A General Bimatrix Games

In general, a 2-player finite game where players have m and n strategies respectively, we may summarize the game as an $m \times n$ "bimatrix"

$$
(A, B) = \begin{array}{c c}
 & \begin{array}{cccc} 1 & 2 & \cdots & n \end{array} \\
\begin{array}{c} 1 \\ 2 \\ \vdots \\ m \end{array} & \begin{pmatrix} (a_{11}, b_{11}) & (a_{12}, b_{12}) & \cdots & (a_{1n}, b_{1n}) \\ (a_{21}, b_{21}) & (a_{22}, b_{22}) & \cdots & (a_{2n}, b_{2n}) \\ \vdots & \vdots & \ddots & \vdots \\ (a_{m1}, b_{m1}) & (a_{m2}, b_{m2}) & \cdots & (a_{mn}, b_{mn}) \end{pmatrix}
\end{array}
$$

Let $I = \{1, \cdots, m\}$ be the set of strategies for Player 1 (the row player) and $J = \{1, \cdots, n\}$ the set of strategies for Player 2 (the column player).

Finite Game in Normal Form

Definition

A finite game Γ in *normal form* may be defined as a triple (N, S, u) where

- $N = \{1, \ldots, n\}$ is the set of players
- $S = S_1 \times \cdots \times S_n$ is the set of joint strategy profiles, S_i is the set of strategies available to player i
- $u : S \to \mathbb{R}^n$ is a utility function mapping each strategy profile to a payoff vector

A Solution for the Game

So is there a solution for the players?

John Nash (1951) proved the existence of equilibrium points for all finite games, provided players allow for randomized strategies.

In a Nash equilibrium, neither player has any incentive to deviate assuming the other player does not.

Definition

A pair of probability vectors (x^*, y^*) is a *Nash equilibrium* for bimatrix game (A, B) if

$$u_1(x, y^*) \leq u_1(x^*, y^*) \ \forall x \in I \text{ and } u_2(x^*, y) \leq u_2(x^*, y^*) \ \forall y \in J$$

Nash Equilibria for "Chicken"

$$\begin{array}{c} \\ D \\ C \end{array} \begin{array}{c} \begin{array}{cc} D & C \end{array} \\ \begin{pmatrix} (0,0) & (7,2) \\ (2,7) & (6,6) \end{pmatrix} \end{array}$$

There are three Nash equilibria for this game:

- Two *pure* equilibria, (D,C) and (C,D)
- One *mixed* equilibrium $\left(\left(\frac{1}{3},\frac{2}{3}\right),\left(\frac{1}{3},\frac{2}{3}\right)\right)$

Formalizing the Coordinated Solution

- A mediator makes a probability distribution over S known to all players.
- It randomly chooses an $s \in S$ according to the distribution and privately informs each player of his component strategy.
- If no player has an incentive to deviate knowing the other players' **conditional distributions**, the probability distribution is self-enforcing.

The Correlated Equilibrium

R. J. Aumann (1974) introduced *correlated equilibrium*.

Definition

A probability distribution μ over S is a *correlated equilibrium* if

$$\forall i \in N, \forall s_i \in S_i, \forall t_i \in S_i \backslash \{s_i\}$$

$$\sum_{s_{-i} \in S_{-i}} \mu(s) \left[u_i(s) - u_i(t_i, s_{-i})\right] \geq 0$$

The linear form

$$h_{s_i,t_i}(\mu) = \sum_{s_{-i} \in S_{-i}} \mu(s) \left[u_i(s) - u_i(t_i, s_{-i})\right]$$

is an *incentive constraint*.

First Proof of the Existence of CE

Aumann proved in 1974 that all games possess a correlated equilibrium.

Proof.

A Nash equilibrium $\sigma = (\sigma_1, \ldots, \sigma_n)$ is a correlated equilibrium where

$$\mu(s) = \prod_{i \in N} \sigma_i(s_i)$$

and every game has a Nash equilibrium. $\square$

The Correlated Equilibrium Set

The set of all correlated equilibria ($\mathcal{C}$) is defined by the following constraints:

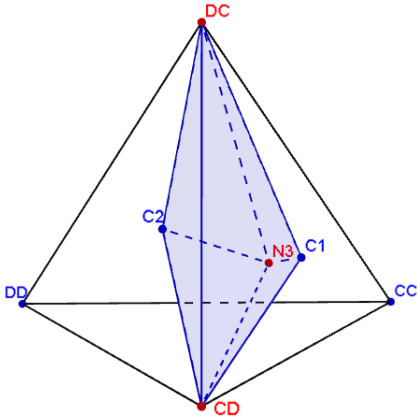
$$\begin{aligned} \mu(s) \geq 0 \;\; \forall s \in S & \quad \text{Non-negativity} \\ \sum_{s \in S} \mu(s) = 1 & \quad \text{Normalization} \\ h_{s_i, t_i}(\mu) \geq 0 \;\; \forall i \in N, s_i \neq t_i \in S & \quad \text{Incentives} \end{aligned}$$

So $\mathcal{C}$ is a polytope (the convex hull of a finite set of extreme points).

Extreme Correlated Equilibria for "Chicken"

$$\begin{array}{cc} & \begin{array}{cc} D & C \end{array} \\ \begin{array}{c} D \\ C \end{array} & \begin{pmatrix} (0,0) & (7,2) \\ (2,7) & (6,6) \end{pmatrix} \end{array}$$

- $\mu_{CD} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$
- $\mu_{DC} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$
- $\mu_{N3} = \begin{bmatrix} 1/9 & 2/9 \\ 2/9 & 4/9 \end{bmatrix}$
- $\mu_{C1} = \begin{bmatrix} 0 & 1/4 \\ 1/4 & 1/2 \end{bmatrix}$
- $\mu_{C2} = \begin{bmatrix} 1/5 & 2/5 \\ 2/5 & 0 \end{bmatrix}$



The Geometric Relationship Between NE and CE

- Hart and Schmeidler (1989) provided a new existence proof using linear methods which does not rely on the existence of Nash equilibria.
- We have already established that the set of Nash equilibria is a subset of $\mathcal{C}$.
- Raghavan and Evangelista (1996) have demonstrated that in bimatrix games the extreme points of so-called Nash sets (maximal convex sets of Nash equilibria) are extreme correlated equilibria.
- Nau et. al. (2004) have shown that in n-player games the Nash equilibria all lie on the (relative) boundary of $\mathcal{C}$.

Model for basic FOA (Brams-Merrill, 1983)

Player I (the minimizer) and Player II (the maximizer) each present a final offer. The arbitrator has an opinion of what he considers fair, and sides with whichever player's offer is closest to the fair settlement.

Assumptions:

- As far as players are concerned, the fair settlement is chosen randomly from a distribution F with density function f.
- F is common knowledge.
- WLOG, the median of the distribution is 0.
- The game is zero-sum.

Optimal Strategies for FOA

Say players choose x_1 and x_2, while the arbitrator chooses ξ.

The payment made by Player I to Player II is

$$K(x_1, x_2|\xi) = \begin{cases} x_1 & \text{if } |x_1 - \xi| < |x_2 - \xi| \\ x_2 & \text{if } |x_1 - \xi| > |x_2 - \xi| \end{cases}$$

If $|x_1 - \xi| = |x_2 - \xi|$, the payment is x_1 or x_2 with equal probability.

Brams-Merrill Theorem (1983)

Theorem

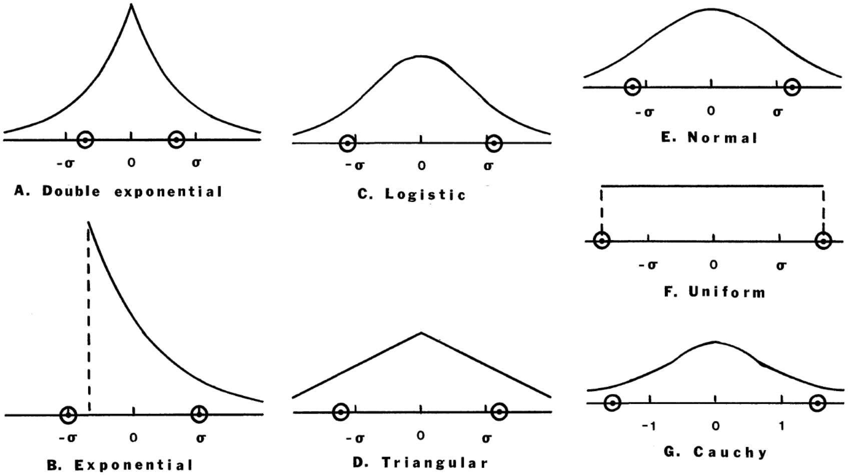
(1) If $f'(0)$ exists and $f(0) > 0$, then locally optimal strategies are

$$x_1^* = -\frac{1}{2f(0)} \quad \textit{and} \quad x_2^* = \frac{1}{2f(0)}.$$

(2) If f is "sufficiently concentrated at the median", then these represent the unique globally optimal strategy pair.

Brams and Merrill also provide a weaker condition for global optimality.
Both Normal and Uniform distributions satisfy the second condition.

Divergence of Global Optimal Pure Strategies



(Brams-Merrill, 1983)

Multiple-Issue FOA

When more than one issue is being arbitrated, two major variants of FOA have been used (Farber, 1980):

- **Issue by Issue:** Each party submits a vector of final offers and the arbitrator is free to compose a compromise by selecting some offers from each party
- **Whole Package:** Both parties submit a vector of final offers and the arbitrator must choose one or the other in its entirety

The Multi-Issue Game Setting

- Players I and II present final-offers $\mathbf{a}, \mathbf{b} \in \mathbb{R}^d$
- Judge selects $\boldsymbol{\xi} \sim F$ as an ideal fair settlement.
- F is common knowledge.
- Judge uses reasonableness function $R(\mathbf{x}, \boldsymbol{\xi}) : \mathbb{R}^d \times \mathbb{R}^d \to \mathbb{R}$ to compare final-offers.
- Game is zero-sum.
- Payoff is

$$K(\mathbf{a}, \mathbf{b} | \boldsymbol{\xi}) = \begin{cases} \sum_i a_i & R(\mathbf{a}, \boldsymbol{\xi}) > R(\mathbf{b}, \boldsymbol{\xi}) \\ \sum_i b_i & R(\mathbf{a}, \boldsymbol{\xi}) < R(\mathbf{b}, \boldsymbol{\xi}) \end{cases}$$

Choice of F

Normal
$\boldsymbol{\xi} \sim \mathcal{N}(\mu, \Sigma)$ (WLOG, assume $\mu = \mathbf{0}$)

Uniform
$\boldsymbol{\xi} \sim \textit{Unif}\left(\times_{j=1}^{d}[-\alpha_j, \alpha_j]\right)$, where $\alpha_j > 0$

Choice of R

Criterion	Reasonableness Function
Net Offer	$R_{NO}(\mathbf{x}, \boldsymbol{\xi}) = -\left\lvert \sum_{j=1}^{d} x_j - \xi_j \right\rvert$
$\mathbf{L}_1$	$R_{L_1}(\mathbf{x}, \boldsymbol{\xi}) = -\sum_{j=1}^{d} \lvert x_j - \xi_j \rvert$
$\mathbf{L}_\infty$	$R_{L_\infty}(\mathbf{x}, \boldsymbol{\xi}) = -\max_j \{\lvert x_j - \xi_j \rvert\}$
$\mathbf{L}_p$	$R_{L_p}(\mathbf{x}, \boldsymbol{\xi}) = -\sum_{j=1}^{d} \lvert x_j - \xi_j \rvert^p$
$\mathbf{L}_2$	$R_{L_2}(\mathbf{x}, \boldsymbol{\xi}) = -\sum_{j=1}^{d} (x_j - \xi_j)^2$
Mahalanobis	$R_M(\mathbf{x}, \boldsymbol{\xi}) = -(\mathbf{x} - \boldsymbol{\xi})' \Sigma^{-1} (\mathbf{x} - \boldsymbol{\xi})$

2NNO

Theorem

Any pure strategies from

$$S_i^* = \left\{ \left(x_1^*, (-1)^i \frac{\sqrt{2\pi(\sigma_x^2 + 2\rho\sigma_x\sigma_y + \sigma_y^2)}}{2} - x_i^* \right) : x_i^* \in \mathbb{R} \right\}$$

$i = 1, 2$ *are independently optimal for Players I and II.*

This is easily found as the game collapses to the one dimensional case.

$2NL_1$, $2NL_\infty$, $2NL_p$

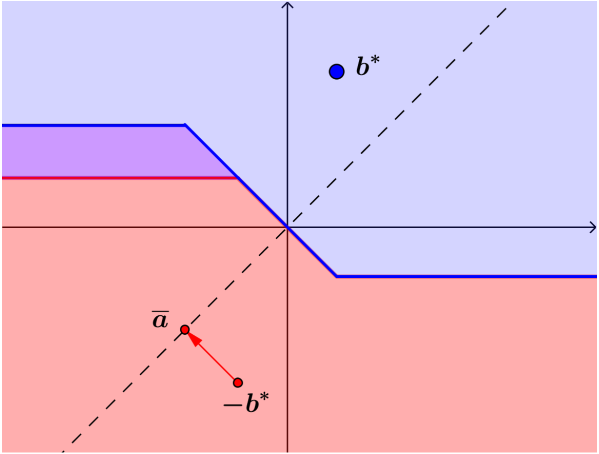
Theorem

In $2NL_1$, $2NL_\infty$ or $2NL_p$ if pure optimal strategies exist for Players $i = 1, 2$ then they are given by

$$(x_i^*, y_i^*) = \left((-1)^i x^*, (-1)^i x^*\right),$$

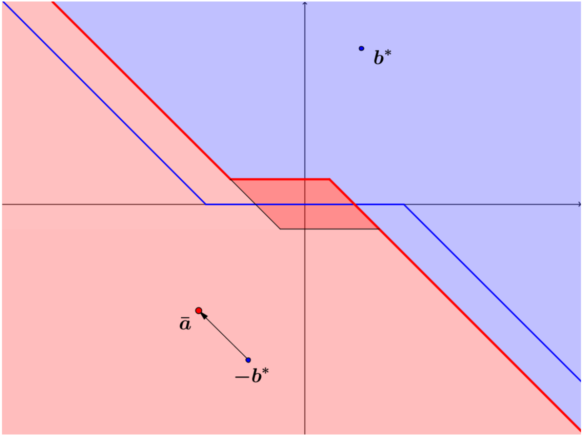
where $x^* = \dfrac{\sqrt{2\pi(\sigma_x^2 + 2\rho\sigma_x\sigma_y + \sigma_y^2)}}{4}$.

$2NL_1$ Proof Sketch



As it is sub-optimal for either player to choose a pure strategy off the line $y = x$, the game reduces to the one-dimensional case.

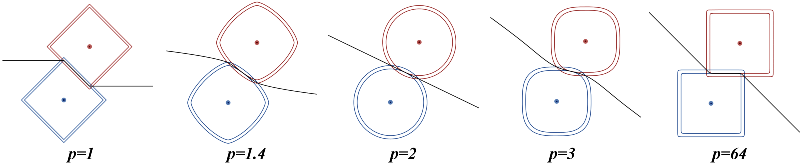
$2NL_\infty$ Proof Sketch



As it is sub-optimal for either player to choose a pure strategy off the line $y = x$, the game reduces to the one-dimensional case.

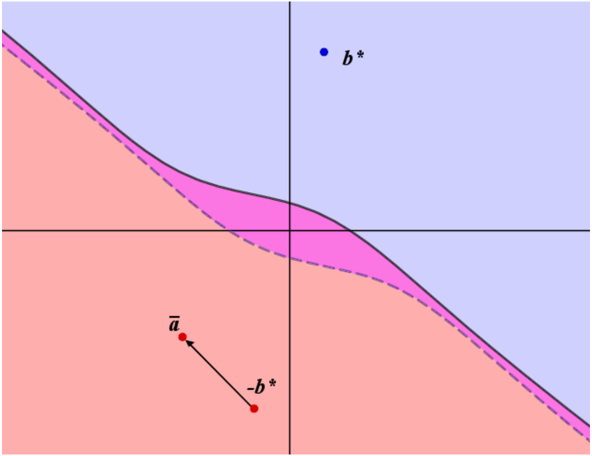
$2NL_p$ Circles

$$D_{L_p}(\mathbf{x}, \mathbf{y}) = \left(\sum_{i=1}^{d} |x_i - y_i|^p \right)^{1/p}$$



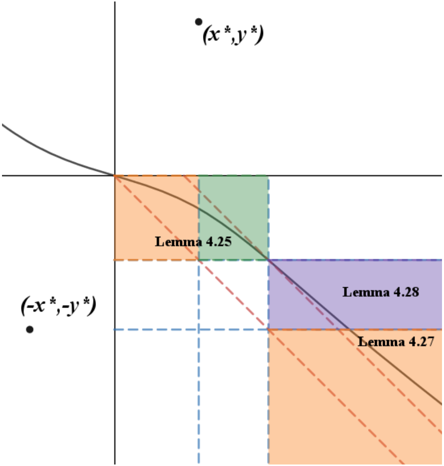
"Circles" and Midset curves under Minkowski metrics (L_p)

$2NL_p$ Proof Sketch



As it is sub-optimal for either player to choose a pure strategy off the line $y = x$, the game reduces to the one-dimensional case.

$2NL_p$ Proof Sketch (cont.)



This is because the midset curve between two such points $-\mathbf{x}^*, \mathbf{x}^*$ in L_p nowhere has a derivative equal to -1.

$2NL_2$ Local Optimality

Theorem

In $2NL_2$, suppose $\rho > \max\left\{-\frac{\sigma_x^2+3\sigma_y^2}{4\sigma_x\sigma_y}, -\frac{3\sigma_x^2+\sigma_y^2}{4\sigma_x\sigma_y}\right\}$. The pure strategy pair in the previous theorem is locally optimal.

Because $Mid_{L_2}[\mathbf{a}, \mathbf{b}]$ is a straight line, we can essentially reduce the dimension of F.

$2NL_2$ Payoff Function

Let $(\xi, \eta) \sim \mathcal{N}(\mathbf{0}, \Sigma)$ be the opinion of the arbitrator, where

$$\Sigma = \begin{bmatrix} \sigma_x^2 & \rho\sigma_x\sigma_y \\ \rho\sigma_x\sigma_y & \sigma_y^2 \end{bmatrix}$$

$C_1(\mathbf{a}, \mathbf{b}) = \{(x, y) | (x_1 - x)^2 + (y_1 - y)^2 < (x_2 - x)^2 + (y_2 - y)^2\}$
and $C_2(\mathbf{a}, \mathbf{b})$ defined similarly.
$P_i = P\big((\xi, \eta) \in C_i(\mathbf{a}, \mathbf{b})\big)$
Assuming $\mathbf{a} \neq \mathbf{b}$, the expected payoff

$$K(\mathbf{a}, \mathbf{b}) = (x_1 + y_1)P_1 + (x_2 + y_2)P_2$$

may be written

$$K(\mathbf{a}, \mathbf{b}) = (x_2 + y_2) + (x_1 + y_1 - x_2 - y_2)P_1.$$

$2NL_2$ Local Optimality Proof Overview cont.

Express P_1 one-dimensionally:

$$
\begin{aligned}
P_1 &= P\big((x_1-\xi)^2+(y_1-\eta)^2<(x_2-\xi)^2+(y_2-\eta)^2\big) && (1)\\
&= P\left((x_2-x_1)\xi+(y_2-y_1)\eta<\frac{x_2^2+y_2^2-x_1^2-y_1^2}{2}\right) && (2)\\
&= P(Z<z) && (3)
\end{aligned}
$$

where

$$
z=\frac{x_2^2+y_2^2-x_1^2-y_1^2}{2\sqrt{(\mathbf{b}-\mathbf{a})^T\Sigma(\mathbf{b}-\mathbf{a})}} \qquad (4)
$$

we may write

$$
K(\mathbf{a},\mathbf{b})=(x_2+y_2)+(x_1+y_1-x_2-y_2)\Phi(z) \qquad (5)
$$

where $\Phi(z)$ is the standard Gaussian cdf.

$2NL_2$ Local Optimality Proof Overview cont.

By solving the system of first-order equations

$$\frac{d}{dx_1}K|_{\mathbf{a}^*,\mathbf{b}^*} = \frac{d}{dy_1}K|_{\mathbf{a}^*,\mathbf{b}^*} = \frac{d}{dx_2}K|_{\mathbf{a}^*,\mathbf{b}^*} = \frac{d}{dy_2}K|_{\mathbf{a}^*,\mathbf{b}^*} = 0$$

we arrive at the unique $\mathbf{a}^*, \mathbf{b}^*$ given in the theorem. It is *straightforward* to verify that the second order condition holds provided

$$\rho > \max\left\{-\frac{\sigma_x^2 + 3\sigma_y^2}{4\sigma_x\sigma_y}, -\frac{3\sigma_x^2 + \sigma_y^2}{4\sigma_x\sigma_y}\right\}.$$

$2NL_2$ Global Optimality

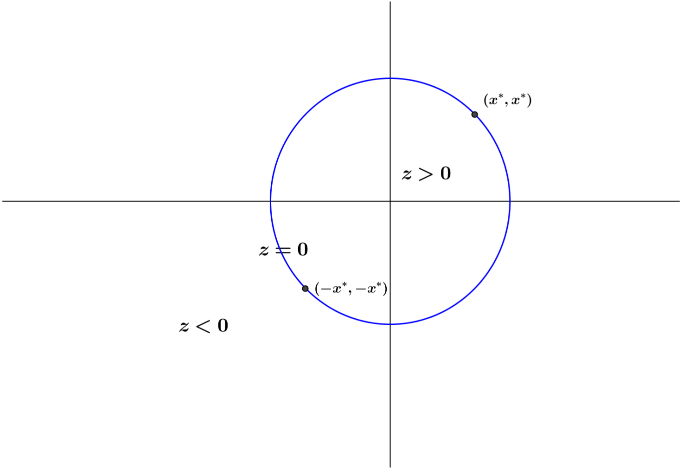
Theorem

If $\rho > 0$, the solution points $\mathbf{a}^*, \mathbf{b}^*$ given in the previous theorem are globally optimal.

In other words, $K(\mathbf{a}, \mathbf{b}^*) \geq 0 \, \forall \mathbf{a} \in \mathbb{R}^2$, with equality only when $\mathbf{a} = \mathbf{a}^*$.

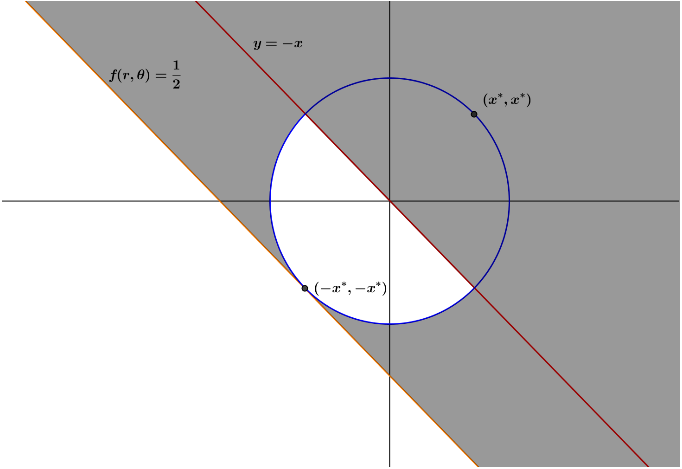
Thus players need not consider mixed strategies. The proof relies on a geometric interpretation of the players' strategies.

$2NL_2$ Global Optimality Proof Overview



$$z = \frac{2x^{*2} - x_1^2 - y_1^2}{2\sqrt{(\mathbf{b}^* - \mathbf{a})^T \Sigma (\mathbf{b}^* - \mathbf{a})}}$$

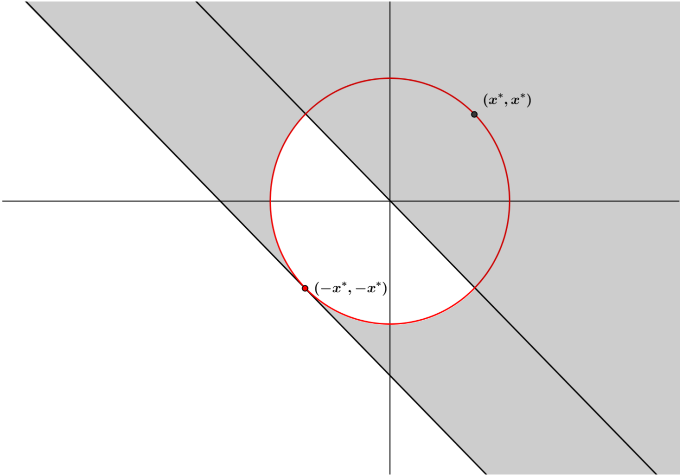
$2NL_2$ Global Optimality: Proof Overview cont.



If $K(\mathbf{a}, \mathbf{b}^*) \leq 0$ then $x_1 + y_1 < 0$ and either

$$x_1^2 + y_1^2 < 2x^{*2} \quad \text{or} \quad x_1 + y_1 \leq -2x^*.$$

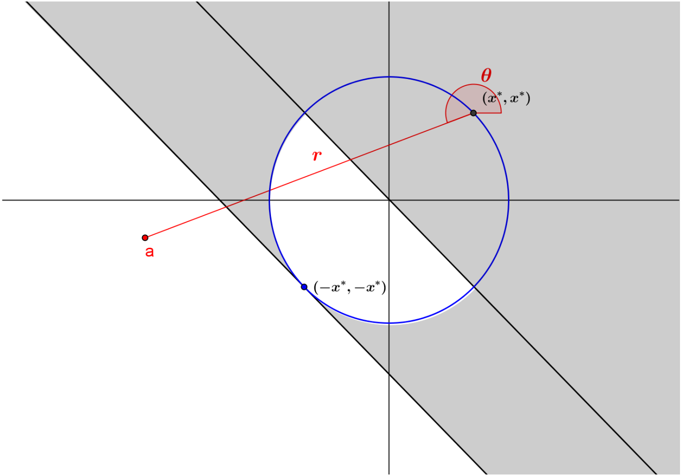
$2NL_2$ Global Optimality: Proof Overview cont.



$$K(\mathbf{a}, \mathbf{b}^*) = 2x^* + \frac{1}{2}(x_1 + y_1 - 2x^*) = 2x^* + x_1 + y_1 \geq 0$$

when $z = 0$, with equality only when $\mathbf{a} = (-x^*, -x^*)$.

$2NL_2$ Global Optimality: Proof Overview cont.



Against Player II's strategy $\mathbf{b}^* = (x^*, x^*)$, any pure strategy $\mathbf{a} = (x_1, y_1)$ may be represented as

$$\mathbf{a}(r, \theta) = (x^* + r\cos\theta, x^* + r\sin\theta).$$

$2NL_2$ Global Optimality: Polar Representation

Letting $t(\theta) = -\cos\theta - \sin\theta$,

$$K(\mathbf{a}, \mathbf{b}^*) = 2x^* - rt(\theta)\Phi(z)$$

So $K < 0$ is equivalent to

$$\phi(z) > \frac{2x^*}{rt(\theta)} = f(r, \theta)$$

$2NL_2$ Global Optimality: Two Tricks

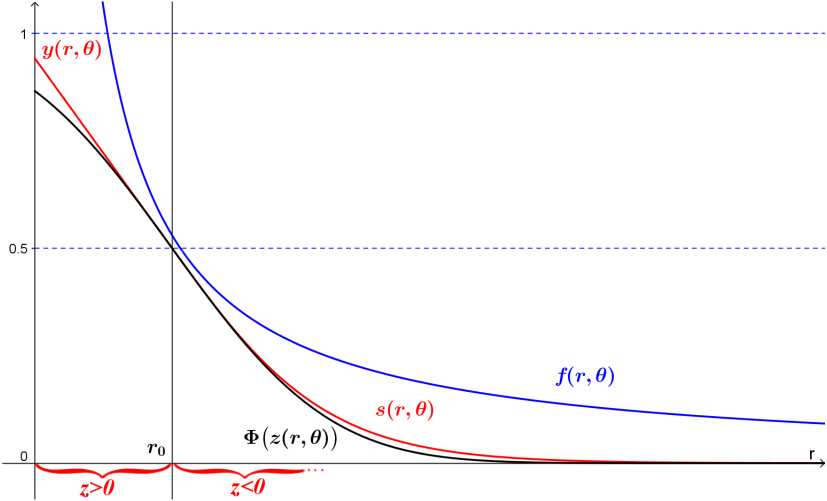
To avoid the difficulties inherent in $\Phi(z)$, we use two tricks:
(1) For $z < 0$, the normal cdf is bounded by the sigmoidal

$$\Phi(z) < \frac{1}{1 + \exp\left(-\sqrt{\frac{8}{\pi}}z\right)}$$

(2) For $z > 0$, by its concavity, $\Phi(z) < y(z)$, the line tangent at $z = 0$

$2NL_2$ Global Optimality: Proof Overview cont.

For fixed $\theta \in \left[\frac{3\pi}{4}, \frac{7\pi}{4}\right]$:



$2UL_2$ - Globally Optimal Pure Strategies

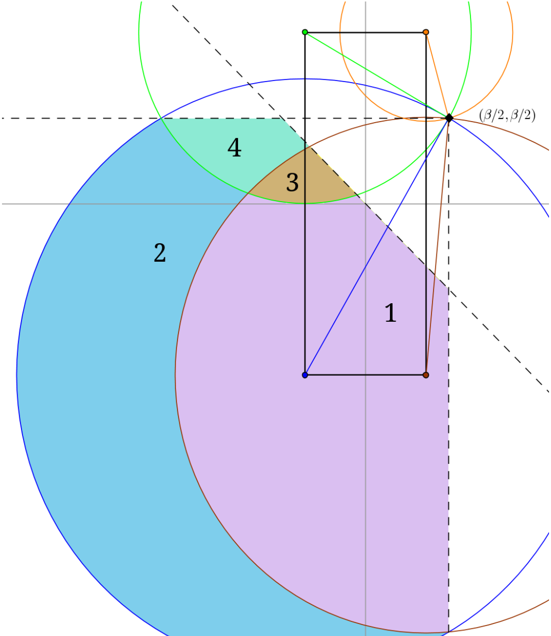
Suppose $\boldsymbol{\xi}$ is drawn uniformly at random from $\Xi := [-\alpha, \alpha] \times [-\beta, \beta]$, where WLOG $0 < \alpha \leq \beta$, and the judge uses the L_2 metric.

Theorem

In $2UL_2$, the strategy pair $\mathbf{a}^* = \left(-\frac{\beta}{2}, -\frac{\beta}{2}\right), \mathbf{b}^* = \left(\frac{\beta}{2}, \frac{\beta}{2}\right)$ is the unique globally optimal strategy pair.

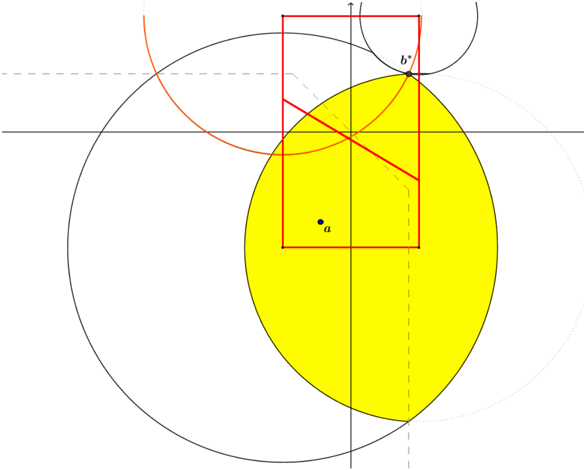
To prove this, we let Player II play $\mathbf{b}^*$ and show that the expected payoff is minimized only when Player I plays $\mathbf{a}^*$.

$2UL_2$ Proof - Cases



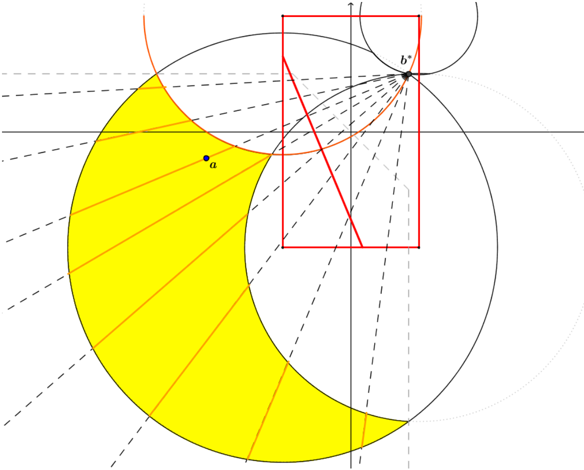
$2UL_2$ Proof Case 1

In the first case, we can show directly that the payoff function is minimized only at $\mathbf{a} = \mathbf{a}^*$, which lies in this region.



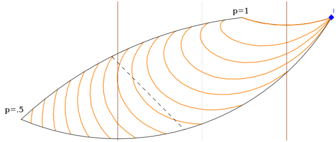
$2UL_2$ Proof Case 2

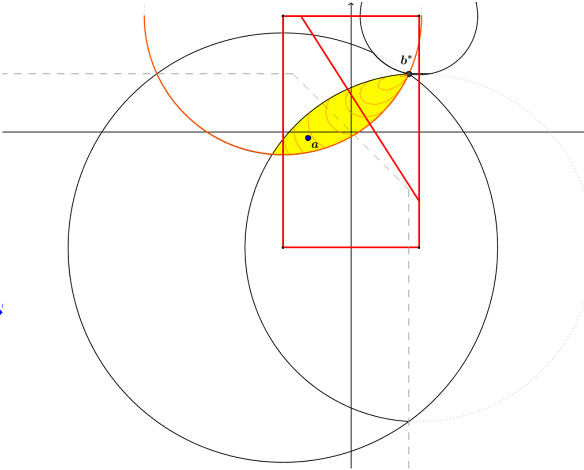
Here we parameterize the strategy of Player I along line segments by slope m and $\lambda \in [0, 1]$, and show that the payoff function is a decreasing function of λ, and on the boundary the payoff is positive.



$2UL_2$ Proof Case 3

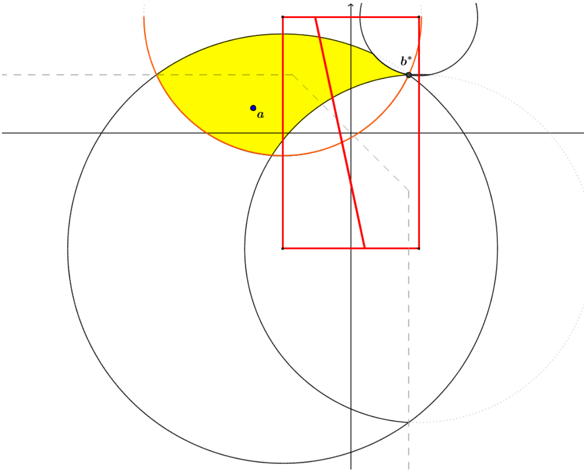
We parameterize the strategy by $p \in [.5, 1]$ (i.e. P_1) and $\bar{x} \in [0, 2\alpha]$ (the length of the upper boundary of C_1) to show there is no local minima.





$2UL_2$ Proof Case 4

The final case is handled directly; it is shown by the first order condition that no minimum to the payoff function exists in this region.



Generalize to N players

A unit must be split between the players. Each player i chooses a vector $\mathbf{x}^i \in \Delta^N$, where $\Delta^N = \{\mathbf{x} | \mathbf{x} \in \mathbb{R}^N, \sum x_i = 1, \mathbf{x} \geq 0\}$. Suppose each player i supplies evidence of strength $\lambda_i \geq 0$ in her favor to the judge. If $\lambda_i = 0$ then the player has supplied no evidence in her favor. Suppose that based on this evidence the judge decides on a fair split of the unit award. Let $\boldsymbol{\xi} = (\xi_1, \xi_2, \ldots, \xi_N)$ be the fair split, where $\boldsymbol{\xi} \in \Delta^N$.

Dirichlet Distribution

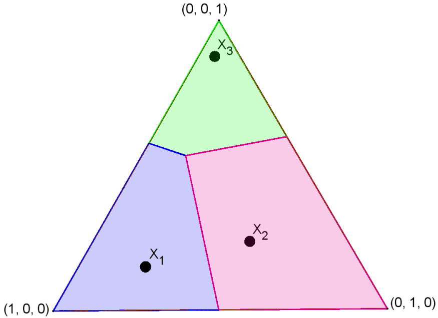
Assume that it is common knowledge among the players that $\boldsymbol{\xi}$ will be drawn from a Dirichlet distribution with parameter $\boldsymbol{\alpha} = (\alpha_1, \ldots, \alpha_N)$, where $\alpha_i = \lambda_i + 1$.. The density function is

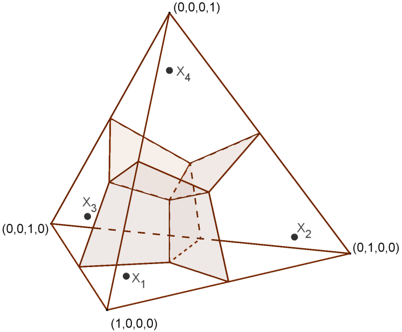
$$f(\mathbf{x}) = \frac{\prod_{i=1}^{N} x_i^{\lambda_i}}{B(\boldsymbol{\alpha})}$$

Where $B(\boldsymbol{\alpha})$ is a normalizing constant.

Voronoi Cells in the Simplex

Given the N final-offers, we may partition Δ^N into N convex Voronoi cells. Call these V_i for $i = 1, \ldots, N$.





If $\lambda_i = 0$ for all players, the probability distribution is uniform over the simplex. In this case, the payoff function is

$$K_i(\mathbf{x}^1, \ldots, \mathbf{x}^N) = (N-1)! \sum_{j=1}^{N} \mathbf{x}_i^j \iint\limits_{V_j} \frac{N!\sqrt{2^N}}{\sqrt{N+1}} dV_j$$

Conjecture: Let $N \geq 3$. Players $2, \ldots, N$ demand β and offer $\frac{1-\beta}{N-1}$ to the opponents. Player 1 determines to demand α. Then P_1 is maximized when Player 1 offers $\frac{1-\alpha}{N-1}$ to each other player.

Theorem

For an N player FOA game where $\boldsymbol{\xi}$ is chosen uniformly at random, assuming the conjecture, a pure equilibrium strategy is for each player to demand $\dfrac{H_{N-1}}{N-1}$ *for himself and offer the remaining portion equally to the other players.*[1]

References

Robert J. Aumann (1974)
Subjectivity and correlation in randomized strategies.
Journal of Mathematical Economics, 1(1):67-96, March 1974.

Fe.S. Evangelista and T.E.S. Raghavan (1996)
A Note on correlated equilibrium.
International Journal of Game Theory, 25(1):35-41.

Sergiu Hart and David Schmeidler (1989)
Existence of correlated equilibria.
Mathematics of Operations Research, 14(1):pp. 18-25.

John Nash (1951)
Non Cooperative Games.
The Annals of Mathematics, 54(2): 286-295, September 1951.

References

Robert Nau, Sabrina Gome Canovas, and Pierre Hansen (2004)
On the geometry of nash equilibria and correlated equilibria.
International Journal of Game Theory, 32(4):443-453.

Brams, S. J., & Merrill III, S. (1983)
Equilibrium strategies for final-offer arbitration: There is no median convergence.
Management Science, 29(8), 927-941.

Stevens, C. M. (1966)
Is compulsory arbitration compatible with bargaining?
Industrial Relations, 5(February): 38-52.

Thank You