Final-Offer Arbitration

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Outline

Negotiation, Threats and Arbitration

An **employer** and **workers' union** enter negotiations over wages.

A **strike** is an expensive alternative for both parties, so the threat to strike is a motivator for agreement.

Should a strike be impossible (for legal or practical reasons) the parties may be contractually obligated to receive a ruling by an judge acting as **arbitrator**. Such arbitration is known as **interest arbitration**.

Conventional Arbitration

Under **conventional arbitration**, the judge takes both sides into consideration and can craft a binding compromise.

Problems:

- Chilling Effect
- Incompatible with bargaining?
- Quality of arbitrated outcomes

Optimal Strategies and Value

In a zero-sum game, where $K(x_1, x_2)$ is received by player II from Player I, a pair of strategies exist (x_1^*, x_2^*) is called an **optimal pair** if

$$K(x_1, x_2^*) \geq v$$

and

$$K(x_1^*, x_2) \leq v$$

for some $v \in \mathbb{R}$. Such a v is called the **value** of the game.

The Chilling Effect Game

The Employer (I, minimizer) and Workers (II, maximizer) submit final offers of a wage increase $x_i \in [0, 1]$ to the judge.

The judge will compromise and rule

$$x = \alpha x_1 + (1 - \alpha) x_2,$$

where $\alpha \in (0, 1)$ fixed but unknown to players.

Regardless of α, the unique optimal strategies are

$$x_1^* = 0, x_2^* = 1.$$

Final-Offer Arbitration

Under **Final-Offer Arbitration [Stevens, 1966]**, the judge examines the final offers of both parties and must **pick one** with no compromise.

Proposed outcomes:

- Combat the Chilling Effect (convergence)
- Motivate Concessions (avoid arbitration)

Certainty of Arbitrator Behavior

Suppose the Employer and Workers are engaged in a FOA game: They choose $x_1, x_2 \in [0, 1]$, and the judge chooses whichever is closer to x.

If players are *certain* of the judge's opinion x then the unique optimal strategy pair is

$$x_1^* = x_2^* = x$$

(Chatterjee, 1981).

A more interesting game must model uncertainty.

Notes on Final-Offer Arbitration

- Beyond the actual cost of arbitration, the risk of uncertainty should (theoretically) motivate players to reach agreement (Stevens 1966).
- Since 1970s, FOA and its variants have been used across the world.
- Under uncertainty, players strike a balance:
  - **extreme** offer: big gain but unlikely win
  - **moderate** offer: smaller gain but more likely win

FOA In Practice

Also known as **Pendulum Arbitration** and **Baseball Arbitration**.

A variant is **MEDLOA** (Mediation with Last Offer Arbitration)

- The Trial of Socrates
- Adopted in Many states (Michigan, Wisconsin 1970s) in the public sector (e.g. police, firefighters)
- Major League Baseball after 1972 strike
- Chile's 1979 Labor Reform
- Railway shipping in Canada

Model for FOA [Brams-Merrill, 1983]

Player I (the minimizer) and Player II (the maximizer) each select a final offer. The arbitrator has an opinion of what he considers fair, and sides with whichever player's offer is closest (in absolute value) to the fair settlement.

Assumptions:

- As far as players are concerned, the fair settlement is chosen randomly from a distribution described by density function f.
- f is common knowledge.
- WLOG, the median of the distribution is 0.
- The game is zero-sum.

Optimal Strategies for FOA

Say players choose x_1 and x_2, while the arbitrator chooses ξ.

The payment made by Player I to Player II is

$$K(x_1, x_2|\xi) = \begin{cases} x_1 & \text{if } |x_1 - \xi| < |x_2 - \xi| \\ x_2 & \text{if } |x_1 - \xi| > |x_2 - \xi| \end{cases}$$

If $|x_1 - \xi| = |x_2 - \xi|$ then the judge may flip a coin to decide between x_1 and x_2, but we shall assume this happens with probability 0.

Minimax Theorem

Theorem

If X_1 and X_2 are compact subsets of Euclidean space and if $K(x_1, x_2)$ is a continuous function of $x_1 \in X_1$ and $x_2 \in X_2$, then the game has a value v, and there exist optimal (mixed) strategies for the players $P_1^* \in \Delta(X_1)$ and $P_2^* \in \Delta(X_2)$ such that

$$K(P_1^*, P_2) \leq v \leq K(P_1, P_2^*)$$

for all $P_1 \in \Delta(X_1)$ and $P_2 \in \Delta(X_2)$.

Here $\Delta(S)$ is the set of all probability distributions over S.

Locally and Globally Optimal Pure Strategies

A pair of pure strategies x_1^*, x_2^* are **locally optimal** if $\exists \epsilon > 0$ such that, for all $x_1 \in N_\epsilon(x_1^*), x_2 \in N_\epsilon(x_2^*)$

$$K(x_1^*, x_2) \leq K(x_1^*, x_2^*) \leq K(x_1, x_2^*)$$

The pair is said to be **globally optimal** if the inequality is true for all $x_1 \in X_1, x_2 \in X_2$.

Brams-Merrill Theorem (1983)

Brams-Merrill provide a stronger result for the single-issue game; **optimal pure strategies** exist under many circumstances.

Theorem

(1) If $f'(0)$ exists and $f(0) > 0$, then locally optimal strategies are

$$x_1^* = -\frac{1}{2f(0)} \quad \textit{and} \quad x_2^* = \frac{1}{2f(0)}.$$

(2) If f is "sufficiently concentrated at the median", then these represent the unique globally optimal strategy pair.

Sufficient Concentration at the Median

A sufficient condition for global optimality is:

$$f(x) \leq f(0) + 4f^2(0)|x| \quad \text{for } |x| \leq \frac{1}{4f(0)},$$

and $\exists c_1, c_2$ with $-\infty \leq c_1 \leq 0 \leq c_2 \leq \infty$ s.t.

$$\begin{aligned} f(x) &\geq f(0)e^{-2f(0)|x|}, \quad c_1 \leq x \leq c_2 \\ f(x) &\leq f(0)e^{-2f(0)|x|}, \quad x \leq c_1 \text{ and } x \geq c_2. \end{aligned}$$

Payoff Function

Consider the expected payment,

$$K(x_1, x_2) = x_1 P(|x_1 - \xi| < |x_2 - \xi|) + x_2 P(|x_1 - \xi| > |x_2 - \xi|)$$

If we assume $x_1 < x_2$,

$$K(x_1, x_2) = x_1 P\left(\xi < \frac{x_1 + x_2}{2}\right) + x_2 P\left(\xi > \frac{x_1 + x_2}{2}\right)$$

Letting $F(x) = P(\xi < x)$, we may write

$$\begin{aligned} K(x_1, x_2) &= x_1 F\left(\frac{x_1 + x_2}{2}\right) + x_2 \left[1 - F\left(\frac{x_1 + x_2}{2}\right)\right] \\ &= (x_1 - x_2) F\left(\frac{x_1 + x_2}{2}\right) + x_2 \end{aligned}$$

Pure Optimal Strategies

If optimal pure strategies x_1^*, x_2^* exist, it must be that

$$K(x_1, x_2^*)$$

is minimized when $x_1 = x_1^*$ and

$$K(x_1^*, x_2)$$

is maximized when $x_2 = x_2^*$.

Suppose that pure optimal strategies do exist, and we derive them as follows:

Strategy optimization

Payoff Function

$$K(x_1, x_2) = (x_1 - x_2)F\left(\frac{x_1 + x_2}{2}\right) + x_2$$

Player I chooses x_1^* to minimize K, so $\frac{d}{dx_1}K(x_1, x_2^*) = 0$ when $x_1 = x_1^*$, that is

$$\frac{x_1^* - x_2^*}{2} f\left(\frac{x_1^* + x_2^*}{2}\right) + F\left(\frac{x_1^* + x_2^*}{2}\right) = 0 \qquad (1)$$

Player II chooses x_2^* to maximize K, so $\frac{d}{dx_2}K(x_1^*, x_2) = 0$ when $x_2 = x_2^*$, that is

$$\frac{x_1^* - x_2^*}{2} f\left(\frac{x_1^* + x_2^*}{2}\right) - F\left(\frac{x_1^* + x_2^*}{2}\right) + 1 = 0 \qquad (2)$$

Deriving pure strategy equilibria

Subtracting (2) from (1) we get

$$F\left(\frac{x_1^* + x_2^*}{2}\right) = \frac{1}{2} \quad \Rightarrow \quad \frac{x_1^* + x_2^*}{2} = 0$$

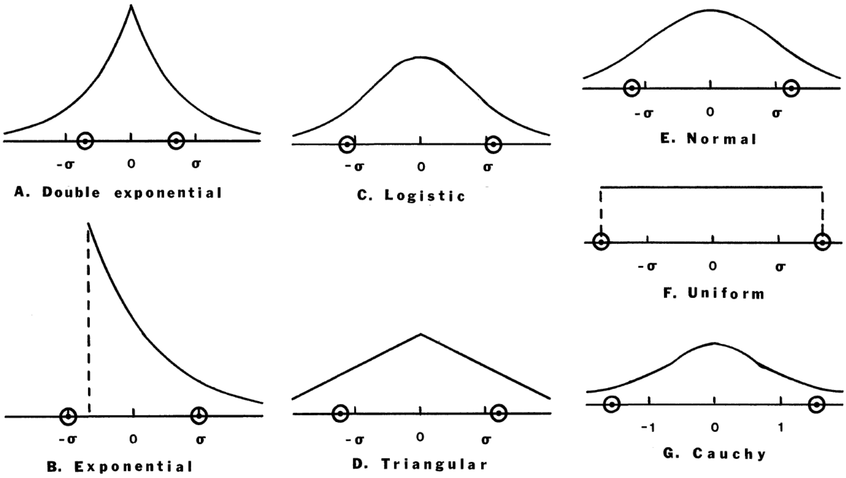
Adding (2) to (1) we get

$$f(0) = \frac{1}{x_2^* - x_1^*} = \frac{1}{2x_2^*} = -\frac{1}{2x_1^*}$$

Thus

$$x_1^* = -\frac{1}{2f(0)}, \quad x_2^* = \frac{1}{2f(0)}$$

Divergence of Global Optimal Pure Strategies



Extended Single-Issue Model

Suppose the arbitrator is of one of two types: $\xi = -1$ or $+1$ with equal probability.

No pure optimal strategies exist. If Players restrict themselves to discrete mixed strategies over integers, many mixed strategies exist. For example:

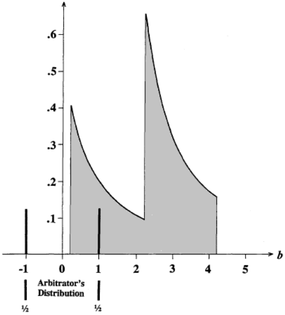
$$x_1 = \begin{cases} -1 & \text{w.p. } \alpha \\ -3 & \text{w.p. } 1-\alpha \end{cases} \quad x_2 = \begin{cases} +1 & \text{w.p. } \beta \\ +3 & \text{w.p. } 1-\beta \end{cases}$$

For any $\alpha, \beta \in [\frac{1}{3}, \frac{1}{2}]$ (Brams 1983).

2 Point Distribution, Cont.

If players can choose any mixed strategy, things get more complicated. Letting $B = \sqrt{5} - 2$, Player II has an optimal mixed strategy (Kilgour, 1994) given by the continuous density function

$$f_2^*(b) = \begin{cases} \frac{(B+1)^{1/2}}{2(b+1)^{3/2}} & B \le b \le B+2 \\ \frac{(B+3)^{1/2}}{2(b-1)^{3/2}} & B+1 < b \le B+4 \end{cases}$$



Asymmetric FOA Variants

- If one player is risk-averse, he tends to make more moderate offers and win more often (Curry 1993, Kilgour 1994).
- The above agrees with empirical evidence (Ashenfelter and Bloom, 1984)
- Dickinson (2006) studied a model where disputants do not share a common belief of the arbitrator's behavior. Optimism leads to the Chilling Effect

Multiple-Issue FOA

When more than one issue is being arbitrated, two major variants of FOA have been used [Farber, 1980]:

- **Issue by Issue:** Each party submits a vector of final offers and the arbitrator is free to compose a compromise by selecting some offers from each party
- **Whole Package:** Both parties submit a vector of final offers and the arbitrator must choose one or the other in its entirety

Extending the Model to Higher Dimensions

To extend the model, a number of questions must be addressed:

- How do players value settlement vectors?
- How does the judge decided which vector is more "reasonable"?
- How do the players model uncertainty?
- How are non-quantitative issues handled?

Dual Issue FOA under Euclidean Distance

Two Issues, additive valuation, zero-sum, Euclidean distance metric.

Theorem

Suppose $\rho > \max\left\{-\frac{\sigma_x^2+3\sigma_y^2}{4\sigma_x\sigma_y}, -\frac{3\sigma_x^2+\sigma_y^2}{4\sigma_x\sigma_y}\right\}$. *Let*

$$x^* = \frac{\sqrt{2\pi(\sigma_x^2 + 2\rho\sigma_x\sigma_y + \sigma_y^2)}}{4}$$

A locally optimal pure strategies pair is

$$\mathbf{a}^* = (-x^*, -x^*), \mathbf{b}^* = (x^*, x^*). \tag{3}$$

If $\rho > 0$, *the strategies are globally optimal.*

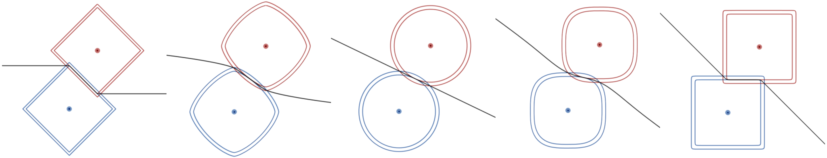
Open Questions

- Alternative Decision Criteria/Distance Measures
- Alternative Package Valuations
- Non-zero sum extension
- Alternative Uncertainty Models
- Generalize to d issues
- Generalize to N players

Alternative Distance Measures: L_p Metrics

L_p distance between points $\mathbf{x}$ and $\mathbf{y}$ is

$$D_{L_p}(\mathbf{x}, \mathbf{y}) = \left(\sum_{i=1}^{d} (x_i - y_i)^p \right)^{1/p}$$



"circles" and midsets between two points, $p = 1, 1.4, 2, 3, 64$

Theorem

For the Dual-Issue FOA using L_p distance ($p \geq 1$), if pure strategies exist then they must be $\mathbf{a}^*, \mathbf{b}^*$ given previously.

Alternative Uncertainty Model: Uniform

Theorem

In a 2-Issue FOA game where $(\xi, \eta) \sim \mathit{Unif}([-\alpha, \alpha] \times [-\beta, \beta])$ *and* $\beta \geq \alpha$*, if pure optimal strategies exist then they are given by*

$$x_1^* = \left(-\frac{\beta}{2}, -\frac{\beta}{2}\right), x_2^* = \left(\frac{\beta}{2}, \frac{\beta}{2}\right)$$

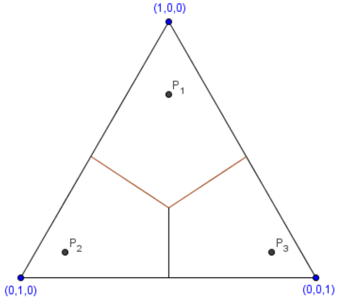
Conjecture: They do exist and are globally optimal.

Generalize to N players

A unit must be split between the players. Players $1, \ldots, N$ choose $P_i \in \Delta^N = \{\mathbf{x} \in \mathbb{R}^N | \sum_{i=1}^N x_i = 1, x_i \geq 0\}$. The judge chooses a fair split $\xi \in \Delta^N$ and chooses whichever P_i is closest to ξ in Euclidean distance.

Theorem

For $N = 3$, demanding $\frac{3}{4}$ for oneself and offering $\frac{1}{8}$ each other player is a pure Nash equilibrium.



References

Ashenfelter, O., and D. E. Bloom (1984)
Models of arbitrator behavior: Theory and Evidence.
American Economic Review, v. 74 (March, 1984):111.

Brams, S. J., & Merrill III, S. (1983)
Equilibrium strategies for final-offer arbitration: There is no median convergence.
Management Science, 29(8), 927-941.

Curry, Amy Farmer Pecornio, Paul (1993)
The use of final offer arbitration as a screening device.
Journal of Conflict Resolution, v37, n4:655.

Dickinson, D. L. (2006)
The chilling effect of optimism: The case of final-offer arbitration.
The Journal of Socio-Economics, 35(1), 17-30.

References

Henry S Farber (1980)
An analysis of final-offer arbitration
Journal of conflict resolution, 24(4), 683-705.

Kilgour, D. M. (1994)
Game-theoretic properties of final-offer arbitration.
Group Decision and Negotiation, 3(3), 285-301.

Stevens, C. M. (1966)
Is compulsory arbitration compatible with bargaining?
Industrial Relations, 5(February): 38-52.

Thank You